APPLIED MECHANICS Reviews

A CRITICAL REVIEW OF THE WORLD LITERATURE IN APPLIED MECHANICS
AND RELATED ENGINEERING SCIENCE

REVS. 1170-1724 VOL. 14, NO. 3 MARCH 1961

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APPLIED MECHANICS Reviews

Published Monthly by THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS at Easton, Pa., and edited by Southwest Research Institute with the co-operation of Linda Hall Library.

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APPLIED MECHANICS REVIEWS, March, 1961, Vol. 14, No. 3. Published Monthly by The American Society of Mechanical Engineers at 20th and Northampton Streets, Easton, Pa., U. S. A. The editorial office is located at the Southwest Research Institute, San Antonio 6, Texas, U. S. A. Headquarters of ASME, 29 West 39th St., New York 18, N. Y., U. S. A. Price $2.50 per copy, $25.00 a year. Changes of address must be received at Society headquarters seven weeks before they are to be effective on the mailing list. Please send old as well as new address. . . . By-laws: The Society shall not be responsible for statements or opinions advanced in papers or printed in its publications (B13, Par. 4). . . . Entered as second-class matter, January 11, 1948, at the Post Office at Easton, Pa., under the Act of March 3, 1897. ©Copyrighted, 1961, by The American Society of Mechanical Engineers.

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APPLIED MECHANICS REVIEWS

VOL. 14, NO. 3 MARCH 1961

RECENT ADVANCES IN THE THEORY OF DRAWING OF THIN SHELLS

W. SZCZEPINSKI

WARSAW TECHNICAL UNIVERSITY (POLITECHNIKA WARSZAWSKA), WARSAW, POLAND

1. INTRODUCTION

The method of drawing is one of the most important methods of plastic forming of thin shells. This method and the method of stretchforming, very similar to it from the point of view of theoretical analysis, are often used in modern industry in plastic forming of shells for airplanes, motor cars, buses, railway cars, etc. Drawing is used in tube production as well as in electronic industry. It may almost be said that there is no branch in the metal industry in which the drawing would be not applied. The theory of drawing is rather well elaborated but for rotationally symmetric shells only. In more general cases when a shell has arbitrary double curvature, effective methods of solution are not yet available because of mathematical difficulties. In recent years there have appeared some papers dealing with this problem and trying to give an analysis of stress and velocity distribution.

2. DRAWING OF ROTATIONALLY SYMMETRIC SHELLS

From the point of view of theoretical analysis, the problem of tube-sinking in which a thin-walled tube is drawn through a conical die, is easiest. Problems of this kind have been investigated by A. Ilyushin (1), A. Nadai (2), G. Sachs and W. Baldwin (4), using approximate assumptions that a material is rigid-plastic without strain-hardening and that the wall thickness of a tube remains constant during the process. By these assumptions the problem becomes statically determinate and can be solved without including the strain rates. Because of small wall thickness b in comparison with the tube diameter, the variations of stresses across the thickness can be neglected and the state of stress can be regarded as a quasi-plane state. The reaction-pressure p between the die and the tube is very small in comparison with the stresses and can be neglected in the yield condition. The equations of equilibrium have the well-known form

$$\frac{d\sigma_\alpha}{dr} + \frac{\sigma_\alpha - \sigma_\theta}{r} = 0; \qquad \frac{\sigma_\alpha}{\rho_\alpha} + \frac{\sigma_\theta}{\rho_\theta} = \frac{p}{b}, \qquad [1]$$

where σ_α is the mean stress parallel to the die, σ_θ the mean circumferential compressive stress. The first equation of equilibrium constitutes, with the yield condition, the system of two equations with two unknown functions $\sigma_\alpha(r)$ and $\sigma_\theta(r)$ and can be solved by elementary integration. For Tresca yield condition, one obtains very simple expressions for the stresses. For Huber-Mises yield condition, however, the solution has a more complicated form than for Tresca condition, but can be used for practical calculations. Having the stresses, we can obtain the pressure p from the second equation of equilibrium in which, for conical die, $\rho_\alpha = \infty$. In a simple manner, the friction forces between the die and the tube can be taken into account. In the first equation of equilibrium there appears a term containing the friction force, which equals $\mu \cdot p$ for unit area, where μ is the friction coefficient. The equations of equilibrium constitute, in common with the yield condition, the set of three equations with three unknown functions $\sigma_\alpha, \sigma_\theta, p$. This set can be solved in a closed form. For practical computations one often uses different approximate methods of solution; some of which are described in (5).

The problem of the tube diameter reduction, the wall thickness variations being taken into account, has been solved by A. Ilyushin (6), H. Swift (3, 7), W. Prager (8–10). In this case the problem is statically indeterminate because in equations of equilibrium [1] there appears a new unknown function of r, namely the wall thickness b. Both equations of equilibrium and the yield condition constitute a set of three equations with four unknown functions σ_α, σ_θ, p, b. The fourth equation can be obtained by using Lévy-Mises relations between the strain rates and stresses and the condition of incompressibility. The solution of this set is rather complex. In (6) it has been integrated by means of the method of finite differences. The results are given in tables and diagrams, very convenient for practical application. In this solution friction forces between the die and the tube were neglected. In (3) these forces were taken into consideration. The Tresca yield condition and the linearized Huber-Mises yield condition were used in (6, 3) and (8–10), respectively. These solutions show that the variation of the wall thickness by drawing is rather small and in extremal case equals 35% in Ilyushin's solution and about 17% in Swift's solution. This variation decreases when the friction forces are taken into account. The results so obtained enable us to neglect the changes in the thickness for practical computations. Recently V. Sokolovsky

published a paper (11) dealing with the same problem. An approximate method of solution has been given by Z. Marciniak (12).

In similar way the problem of deep-drawing of cylindrical cups can be solved. In such a process a thin circular blank is formed into a cup. Usually the blank is placed between a die plate and a blank holder. The cup is formed by a punch forced down and by a draw ring made in the middle of a die plate. The formulas for stress distribution within the blank have a very simple form, provided the changes of thickness are disregarded. The solution, when the thickness variation is taken into account, has been given by R. Hill (13, 14) and G. Sachs (15, 16).

Interesting results were obtained by Z. Marciniak (17), who has investigated a shape of parallel cross sections of shells formed by special methods of drawing in which the material of a shell has no contact area with a die. The particular process where the wall thickness of a cup is reduced while the internal diameter is kept constant was investigated by R. Hill (18, 19). He pointed out that, for the small thickness in comparison with the cup diameter, the problem can be treated as one of plane strain.

As can be seen from the above brief review, the problem of drawing of rotationally symmetric shells has been rather thoroughly investigated although, due to well-known difficulties appearing in these solutions, the strain-hardening of the material was neglected.

3. DRAWING OF SHELLS WITH ARBITRARY DOUBLE CURVATURE

The problem of plastic forming of thin shells with arbitrary double curvature has been mathematically formulated by A. Ilyushin (20). In this paper the general equations of equilibrium have been formulated, the strain-hardening of material, changes in wall thickness, friction forces between the material and the die and inertial forces being taken into consideration. Because of small wall thickness, the variations of stresses and velocities through the thickness were neglected and the mean stresses and velocities only were considered. Three kinds of methods of plastic forming of thin shells were distinguished. The first occurs when the material is pressed between the die and the punch. This method is well known as forging. The pressure, normal to the shell surface, has a value close to the yield point of the material and, of course, it can not be neglected in the yield condition, as can be done for drawing. It has been shown that for the pressing of the thin layer of a metal between two plane rigid plates the distribution of the pressure can be obtained by using the sand hill analogy. Piling sand on a horizontal base that has a shape of the contact surface, we obtain a sand hill, whose ordinates are proportional to the differences $(p - \sigma_{p1})$, where p is the pressure on the contact surface, σ_{p1} the yield point of the material. The second kind of process of plastic forming occurs when the material has the contact area with a die on one surface only. We have this situation in drawing and stretchforming. To solve the problem it is sufficient to find 5 unknowns at any point of the shell, namely: p, the pressure between the die and the material; u and v, the velocities of flow in the coordinate directions; b, the wall thickness; and λ, the coefficient of deformation. Three equations of equilibrium, the condition of incompressibility and the condition for λ constitute a system of five equations with above five unknown functions. Having u, v and λ one can obtain the strain rates and the stresses at any point. Unfortunately, this system is very complex and the author has not presented any method of solution. The third and last kind of plastic forming method considered by Ilyushin is that where the material has no contact area with the die.

The next step in the theory of drawing was made by A. Tomlenov (21, 22). He pointed out that by drawing, where the principal stresses have different signs, the slip lines on the surface of the shell can be constructed. The relations between the stresses along these lines have been derived. The author proposed to apply the approximate methods of the theory of plasticity in order to construct the net of slip lines, but he has not mentioned which of them; it may be a graphical one. It is difficult to say something about the practical usefulness of the proposed method since no example was presented.

It was shown (23, 24) that, based on ideas given in foregoing papers, an effective method of solution can be obtained assuming that the wall thickness remains constant during the process and that it has the same value at any point of the shell. A rigid-plastic material without strain-hardening was assumed. The method proposed is very similar to that used in the plane stress analysis (25). The solution takes the simplest form when the friction forces between the die and the sheet are disregarded. In this case the problem becomes statically determinate and can be solved in terms of static analysis only, the velocity equations being uncoupled. The three stress components and the reaction-pressure p between the die and the material must be determined at any point of the shell. This pressure must be distributed in such a manner that the material is plastic at any point. Assuming an orthogonal system of coordinates $\alpha\beta$, the equations of equilibrium can be written in the form

$$\left.\begin{aligned} (\sigma_\alpha - \sigma_\beta)\frac{\partial B}{\partial \alpha} + B\frac{\partial \sigma_\alpha}{\partial \alpha} + 2\tau\frac{\partial A}{\partial \beta} + A\frac{\partial \tau}{\partial \beta} = 0,\\ (\sigma_\beta - \sigma_\alpha)\frac{\partial A}{\partial \beta} + A\frac{\partial \sigma_\beta}{\partial \beta} + 2\tau\frac{\partial \beta}{\partial \alpha} + B\frac{\partial \tau}{\partial \alpha} = 0,\\ \frac{\sigma_\alpha}{\rho_\alpha} + \frac{\sigma_\beta}{\rho_\beta} = \frac{p}{b}, \end{aligned}\right\} \quad [2]$$

where A and B are the coefficients of the first fundamental form. Moreover, the stress components must satisfy the assumed yield condition. The problem has been solved for both Tresca and Huber-Mises yield conditions. To explain the method of solution, let us assume the latter, which can be written in the form

$$\frac{1}{3}(\sigma_\alpha^2 - \sigma_\alpha\sigma_\beta + \sigma_\beta^2) + \tau^2 = k^2 = \frac{1}{3}\sigma_{p1}^2. \quad [3]$$

This condition in common with two first equations of equilibrium [2] constitutes a system of three equations with three unknown functions σ_α, σ_β, τ. The pressure p can be obtained from the third equation of equilibrium [2] upon determining the stresses. Using the expressions (25)

$$\frac{\sigma_\beta}{\sigma_\alpha} = k(\sqrt{3}\cos\omega \pm \sin\omega\cos 2\varphi), \qquad \tau = k\sin\omega\sin 2\varphi,$$

the yield condition [3] is identically satisfied. Substituting these expressions into two first Eqs. [2], we obtain the set of two quasi-linear differential equations with two unknown functions ω and φ. This set can be solved by the method of characteristics. The equations of the characteristics are

$$\left.\begin{aligned} &\text{the first family} — \frac{d\alpha}{d\beta} = \frac{B}{A}tg(\varphi + \psi);\\ &\qquad d\xi = \frac{1}{A}\frac{\partial B}{\partial \alpha}d\beta - \frac{1}{B}\frac{\partial A}{\partial \beta}d\alpha;\\ &\text{the second family} — \frac{d\alpha}{d\beta} = \frac{B}{A}tg(\varphi - \psi);\\ &\qquad d\eta = \frac{1}{A}\frac{\partial B}{\partial \alpha}d\beta - \frac{1}{B}\frac{\partial A}{\partial \beta}d\alpha; \end{aligned}\right\} \quad [4]$$

where ψ is the auxiliary function of ω, and ξ, η the auxiliary functions of ω and φ introduced in order to simplify Eqs. [4]. These characteristics are not always real; in other words, the problem is not always hyperbolic. The range in which it is hyperbolic is marked by heavy lines at Fig. 1. It can easily be shown that by drawing (Fig. 2c) and stretchforming of shells with negative Gauss curvature (Fig. 2a) the problem is hyperbolic. For other cases (Fig. 2b d) regions can appear where it is elliptic. In these regions the method of characteristics can not, of course, be applied. The solution of particular problems consists in solving the boundary-value problems for Eqs. [4], such as the Cauchy problem, the characteristic problem, and different mixed problems. However, in many practical cases it is impossible to formulate exact boundary conditions and the solution can be obtained only by approximate assumptions. Fig. 3 shows us the net of characteristics obtained for an example of drawing of the cup with oval cross section. The computations were carried out by means of the method of finite differences.

It has been shown that for Tresca yield condition, as in plane stress analysis, the characteristics constitute simultaneously the slip lines.

In many cases of shell drawing the conditions of flow are those of steady motion. For these cases the velocities of flow of the material can easily be obtained. Using the well-known Lévy-Mises relations between strain rates and stresses and expressing the strain rates in terms of velocities, one obtains a set of two differential equations with two unknown velocities v_α and v_β in directions of α and β, respectively. If the stress distribution is determined on the basis of the Huber-Mises yield condition [3], the equations of characteristics for the velocities are the same as for the stresses. Having the differential relations, which must be satisfied along characteristics, we can obtain the velocities at any point, solving boundary-value problems by means of the method of finite differences.

The friction forces between the sheet and the die can be taken into account in a simple way (24). It is assumed that the friction coefficient μ is constant over the entire region of contract and that the friction force per unit area is $T = \mu \cdot p$, where p is, as before, the pressure of the sheet on the die. It is assumed, in addition, that the vector of the friction force acting on the sheet is opposite to that of velocity at the same point. The friction force components T_α and T_β in α and β directions respectively can now be expressed

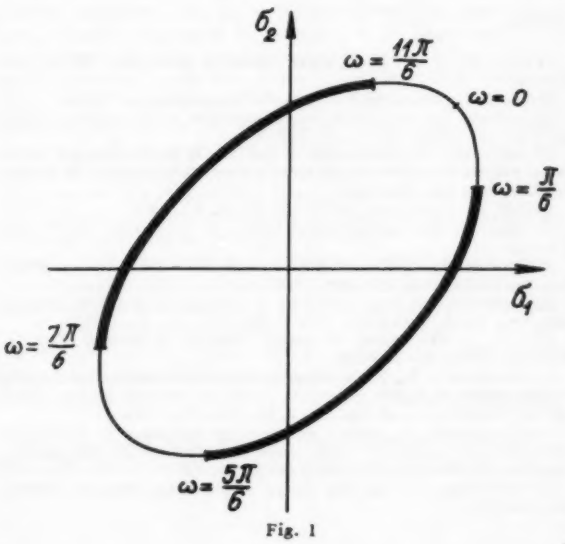

Fig. 1

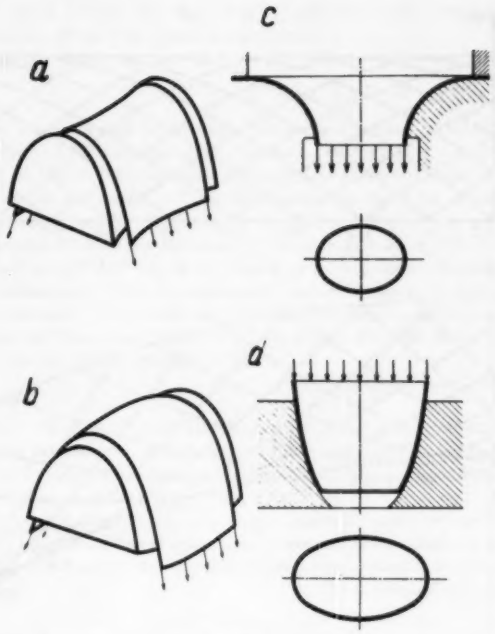

Fig. 2

in terms of velocities $T_\alpha = \mu p \frac{v_\alpha}{v}$, $T_\beta = \mu p \frac{v_\beta}{v}$, where $v = \sqrt{v_\alpha^2 + v_\beta^2}$ – the velocity of flow at the point considered. The problem is statically indeterminate because in the two first equations of equilibrium [2] there appear the expressions for friction forces containing velocities. Using the same method of solution as before, one obtains the same differential equations of characteristics

$$\frac{d\alpha}{d\beta} = \frac{B}{A} tg\,(\varphi \pm \psi).$$

The differential relations that must be satisfied along characteristics have now a more complicated form than previously and contain the velocities. Two cases of boundary-value problems can now be distinguished. If simultaneous values of stresses and velocities are given at one edge, the net of characteristics can be computed at once in the final form, starting from this edge. All the unknowns at any point of the net can be obtained by means of the method of finite differences, by computing successive approximations. The problem is more complicated where stresses are given at one edge and velocities at the other one. However, the solution can also be obtained in this case by the method of successive approximations. First, starting from the edge with given stresses, we can compute the first approximation for stresses at any point, the friction forces being neglected. Having found stresses at the edge with given velocities, we can start to compute the first approximation of velocities, friction forces being taken into account. This procedure ought to be repeated until the difference between two successive approximations becomes sufficiently small. It has been shown that for drawing the second or third approximations are sufficient, because of small changes of velocities caused by friction forces.

4. CONCLUSIONS

The theory of drawing of shells with arbitrary double curvature presented here represents the first step only in the con-

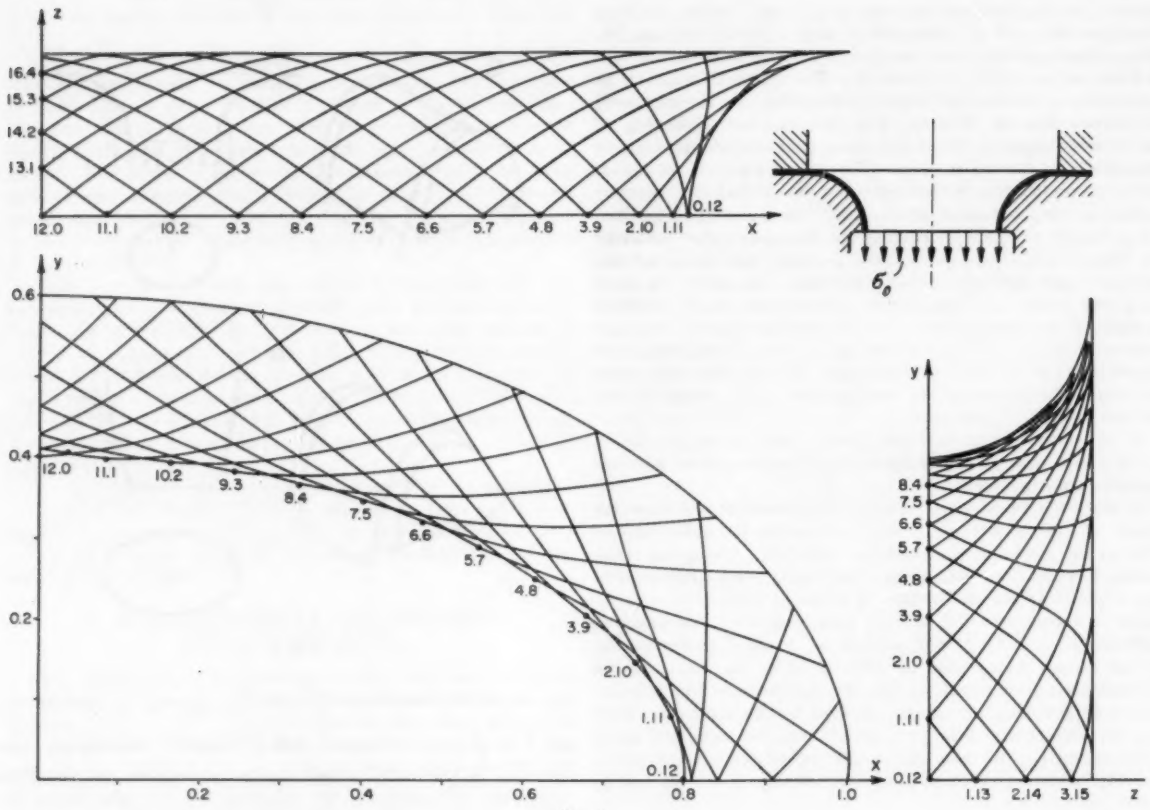

Fig. 3

sideration of practical problems. The main difficulty is to formulate correct boundary conditions, because the curvilinear part of the sheet, which has contact with the draw ring, is joined with the plane part of the blank and with the cylindrical part of the cup. In many cases when the boundary values can be easily obtained, as for example by stretchforming (Fig. 2a b), there occur discontinuity lines for stresses. These discontinuities are of the kind impossible from the point of view of equilibrium and the solution so obtained is not correct.

It would seem that the next steps in the theory of drawing will give us, first of all, solutions of particular problems by above-mentioned assumptions. It is possible that future investigations will take changes of the wall thickness into account.

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Physics, General

(See also Rev. 1655)

Book—1170. Troskolanski, A. T., Matczynski, F., and Stabrowska, I., editors, Small encyclopedia of technics [Mala encyklopedia techniki], Warszawa, Panstwowe Wydawnictwo Naukowe, 1960, 1162 pp.

A concise manual of information on wide range of modern engineering contains chapters on general mechanics, materials, mining, energy production, metallurgy, chemical engineering, chemical industry, machines, electrical engineering, constructions, communication, paper, textile and food industry, rearmament organization, and history of engineering. All chapters are short, but condensed information is quite close to the present level of knowledge and progress. This book can be of great value for general education.

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Analytical Methods in Applied Mechanics

(See also Revs. 1182, 1183, 1200, 1201, 1208, 1211, 1223, 1256, 1266, 1268, 1276, 1299, 1302, 1309, 1361, 1410, 1413, 1455, 1470, 1477, 1531, 1535, 1593, 1599, 1617, 1672, 1675)

1171. Semarne, H. M., New direct method of solution of a system of simultaneous linear equations, *SIAM Rev.* **1,** 1, 53–54, Jan. 1959.

The classical method of solving a system of n linear equations demands the calculation of $n+1$ determinants and the division of n of them by the $(n+1)^{th}$, all in all $2n+1$ operations. The method proposed by present author consists in orthogonalizing a determinant of $n+1$ rows and columns and dividing the n first terms of the last row of the orthogonalized determinant by the last terms, totally $2n$ operations. It is easily seen that the result will be the same as in the classical method. It is necessary to know the capacity of computers in order to decide if the proposed method needs less time to realize the mentioned operations. As a matter of fact, it reduces the problem to the form of only one determinant.

J. R. Delacre, Argentina

1172. Kadner, H., Investigation of the collocation method (in German), *ZAMM* **40,** 1/3, 99–113, Jan./Mar. 1960.

This paper deals with approximate solutions of boundary-value problems associated with linear ordinary differential equations, using a method invented by R. A. Frazer, W. P. Jones and S. W. Skan ["Approximations to functions and to the solutions of differential equations," Aero. Res. Counc. Lond., Rep. Mem. 1799 (2913), 1937]. No investigation referring to best approximation of solutions, convergence and bounding of errors has been published since then. These matters are elucidated here. Results are compared with those of square-error method. It follows that the collocation method is a special form of a more general procedure of round-off errors. Zeros of Legendre polynomials are preferable as collocation values under general conditions.

Author uses simple analytical means and obtains good results, as is demonstrated by two completely numerically worked out problems. The first one corresponds as D.E.: $((2-x^2)y'')'' + 40\,y = 2 - x^2$, with boundary conditions $y(\pm 1) = y''(\pm 1) = 0$; the second D.E. is $x^3y''' + 4x^2y'' + 3xy' + \frac{1}{2}y = x$, with $y(1) = y'(2) = y''(1) + y'(1) = 0$. Extensive bibliography is furnished.

O. Cerceau, Venezuela

1173. Ogurtsov, A. I., The stability of solutions of two nonlinear differential equations of the third and fourth orders, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **23,** 1, 247–251, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Consider the equation

$$\xi^{(iv)} + f(\xi''') + c\,\xi'' + b\,\xi' + a\,\xi = 0 \qquad [1]$$

where primes denote derivatives with respect to time. Author proves the following theorems. Theorem 1. If $a > 0$, $b > 0$, the function $f(\eta)$ is differentiable, $f(0) = 0$, and $df/d\eta > a/b$ for all η, then the zero solution of [1] is asymptotically stable for arbitrary initial perturbations. Theorem 2. If $a > 0$, $b > 0$, $f(0) = 0$, and the derivative $df/d\eta$ of the function $f(\eta)$ is such that $df/d\eta > 0$, $bc\,(df/d\eta) - b^2 - a\,(df/d\eta)^2 > 0$ for all values of the argument, then the zero solution of [1] is asymptotically stable for arbitrary initial perturbations. The proofs are given by constructing appropriate Liapunov V-functions [see also Pliss, V. A., *Dokladi Akad. Nauk SSSR (N.S.)* **111,** 1178–1180, 1956].

E. Leimanis, Canada

1174. Herman, R., and Radok, J. R. M., High order correct difference schemes for anisotropic parabolic equations, AFOSR TN 60-1304 (Polyt. Inst. Brooklyn, Dept. Aero. Engng. Appl. Mech., PIBAL Rep. no. 581), 7 pp., Oct. 1960.

Difference schemes for anisotropic parabolic equations are deduced for given order of truncation errors and their relationship to similar schemes for the one-dimensional heat equation is demonstrated.

From authors' summary

1175. Kuzmak, G. E., Asymptotic solutions of nonlinear second-order differential equations with variable coefficients, *Appl. Math. Mech. (Prikl. Mat. Mekh.)*, **23,** 3, 730–744, 1959. (Pergamon Press, Inc., 122 E. 55th St., New York 22, N. Y.)

Paper is a continuation of author's previous works. Approximate calculation procedures based on N. M. Krylov and N. N. Bogoliubov's method are indicated for the solutions of the equation

$$\frac{d^2y}{dt^2} + \varepsilon f(\tau, y)\frac{dy}{dt} + F(\tau, y) = 0$$

where $\tau = \varepsilon t$, ε being a small parameter. Only the cases in which the equation has, in a first approximation, a family of periodic solutions are considered.

The solutions are given with a precision of the order ε^2 by a function of the form

$$y(t) = y_0(\tau, \omega) + \varepsilon\, y_1(\tau, \omega)$$

where $d\omega/dt = \varphi(\tau)$ is a function which differs from zero and is positive in the considered range, while y_0 and y_1 may be determined from auxiliary equations out of which one of the form $\varphi^2(\tau)\dfrac{\partial^2 y_0}{\partial \omega^2} + F(\tau, y_0) = 0$ is termed "standard" equation.

Further, the particular case in which the solution of the standard equation is expressible by means of elliptic functions of Jacobi is discussed. When the solution of this equation cannot be expressed in terms of special functions, an approximate procedure is indicated.

T. Hacker, Roumania

1176. Barbashin, B. A., and Tabueva, V. A., The conditions for the existence of limit cycles, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **23,** 5, 1185–1197, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

A qualitative investigation of a nonlinear differential equation

$$\ddot{x} + R(x, \dot{x}) + f(x) = 0, \qquad [1]$$

where $f(x) = f_1(x)$ for $\dot{x} > 0$, $f(x) = f_2(x)$ for $\dot{x} \le 0$, is carried out. Conditions are imposed on the functions $R(x, \dot{x})$, $f_1(x)$ and $f_2(x)$, such that equation [1] describes, in general, the motion of a particle along a certain closed curve, subject to the action of a pushing

force. It is shown that under certain conditions the presence of such a force assures the existence of at least one stable limit cycle of equation [1]. E. Leimanis, Canada

1177. Henderson, J. C. de C., Topological aspects of structural linear analysis, *Aircr. Engng.* **32,** 375, 137–141, May 1960.

The selection of a system of cut releases which would render a continuous skeletal structure statically determinate is governed principally by the topological and geometric characteristics of the structure, and to a lesser extent by its material properties. Only the topological aspects are considered here, the other factors being purposely excluded. It is shown how knowledge of the topology of the structure can be exploited to improve the conditioning of the associated equations of compatability. Owing to difficulties inherent in the design of any automatic procedure for the reduction to a canonical form, a method of condensation has been evolved for complex multi-member structures. In this way the large-scale features of the structure are taken into account without neglecting the effect of local characteristics.

From author's summary by Y. C. Fung, USA

1178. Tanimura, M., On the solution of some mixed boundary problems: X, The problem of an L-shaped domain, *Technol. Rep. Osaka Univ.* **10,** 75–84, Jan. 1960.

The method of solving differential equations subject to mixed boundary conditions by series of repeated contour integrals was extended to some cases of an L-shaped domain, introducing double series of repeated contour integrals.

From author's summary

Computing Methods and Computers

(See also Revs. 1175, 1295, 1412, 1454, 1593, 1683, 1714)

1179. Lehmann, N. J., An error estimate for the Ritz method for nonhomogeneous boundary-value problems (in German), *Numerische Mathematik* **2,** 2, 60–66, Mar. 1960.

Improvements in the evaluation of the error by the approximate solution of the boundary-value problem $y'' - q(x)y = r(x)$, $y(0) = y(1) = 0$ by the Ritz-Galerkin method [AMR **5**(1952), Rev. 970; AMR **10**(1957), Rev. 957] are given in the case of piecewise continuous coefficients q, r. R. Conti, Italy

1180. Borsch-Supan, W., Remarks on the error estimate for the Ritz-Galerkin method after Kryloff (in German), *Numerische Mathematik* **2,** 2, 79–83, Mar. 1960.

Further improvements in the evaluation of the error for the same problem considered by N. J. Lehmann [See preceding review] are obtained in the case of coefficients q, r twice differentiable, with second derivatives in $L_2[0, 1]$. R. Conti, Italy

1181. Bialy, H., Iterative solution of a matrix equation (in German), *ZAMM* **40,** 1/3, 130–132, Jan./Mar. 1960.

Author shows how H. Buckner's iterative method for solving integral equations [*Math. Nachr., Berlin* **2,** 304–313, (1949)] can be ingeniously and usefully applied to solution of matrix equation $AX = B$ if A is hermitean; if $B = 1$, solution yields inverse of A. Though method converges less rapidly than that of G. Schultz [*ZAMM* **13**, pp. 57–59, (1933)], it has advantage of involving only one matrix multiplication per iteration (instead of Schultz' two) and of avoiding labor of constructing good initial approximation. Outline of numerical example shows inversion of 3×3 matrix with elements between -10 and 10 is accomplished correctly to 6 decimals in 21 simple iterations. Reviewer believes omission of $(-1)^{\nu-1}$ suggested in § 2 would spoil preceding proof of convergence.

B. Hoffmann, USA

1182. Wetterling, W., A theorem on bounds of Kryloff-Bogoliubov for general eigenvalue problems in ordinary differential equations (in German), *Numerische Mathematik* **2,** 1, 18–21, Jan. 1960.

A theorem on the location of the eigenvalues of a general two-point boundary-value problem for a self-adjoint differential equation of arbitrary order is extended from a case considered by Krylov and Bogoliubov [see L. Collatz, AMR **3**(1950), Rev. 2152] to a case considered by E. Kamke [*Math. Z.* **46,** p. 231, 1940].

R. Conti, Italy

1183. Talbot, A., The evaluation of integrals of products of linear system responses, Parts 1 and 2, *Quart. J. Mech. Appl. Math.* **12,** 4, 488–520, Nov. 1959.

Part I. Paper gives numerical procedure for the integrals $\int^{\infty} xy\,dt$ and $\int^{\infty} x^2\,dt$ when $x(t)$ and $y(t)$ have rational Laplace transforms. Extension to moment integrals $\int^{\infty} t^r xy\,dt$ and $\int^{\infty} t^r x^2 d$, $r = 1, 2, \ldots$ is also indicated. The integrals are expressed as one of the unknowns in a set of simultaneous equations whose matrix involves the coefficients in the Laplace transforms x and y. The labor of equation solving is reduced by exploiting the special form of the matrix. Numerical illustrations are given.

Part II. In Part I it was shown how the problem of evaluating such integrals as $\int^{\infty} t^r xy\,dt$ could be solved by means of a set of linear equations. In Part II the use of continued fractions is shown to be more economical. B. Langefors, Sweden

1184. Radok, J. R. M., On asymptotic numerical methods for parabolic equations, AFOSR TN 58-1072 (Polyt. Inst. Brooklyn, Dept. Aero. Engng. Appl. Mech. Rep. 485; ASTIA AD 207 241), 14 pp., Nov. 1958.

Using diffusion equation as example author suggests an explicit finite difference recurrence formula whose coefficients depend on the particular solution being marched out. Advantage of the method is that the time interval can be increased without instability as the spatial distribution smooths out.

S. H. Crandall, USA

1185. Wachspress, E. L., and Habetler, G. J., An alternating-direction-implicit iteration technique, *J. Soc. Indust. Appl. Math.* **8,** 2, 403–424, June 1960.

In solving partial differential equations of elliptic type by means of electronic digital computers, the alternating-direction method first developed by Peaceman and Rachford [*J. Soc. Indust. Appl. Math.* **3,** 28–41, 1955] has surprising efficiency.

Paper presents the theory and experience (in neutron diffusion calculations) on some improvements incorporated in newer version of one such code. Main improvements are: (1) Conditioning of matrices—the transformation of equations in such a manner as to get matrices which are mutually commutative; (2) automatic calculation of iteration parameters, based on minimax principle; (3) new computation scheme, which requires storage of only one quantity in fast memory and smaller number of arithmetic operations.

Reviewer believes this is a good example of the merits of interplay between theory and practice in the art of high-speed computing. S. Moriguti, Japan

1186. Ernst, H. M., Stein, M., Hancock, H. L., and Heindish, B., A digital computer code translator for use with the Univac high-speed printer, David W. Taylor Mod. Basin Rep. 1403, 43 pp., July 1960.

This report summarizes the design, development, and construction of a transistorized translator to permit standard IBM-705 Com-

puter output tapes to be processed on a Remington Rand UNIVAC High-Speed Printer. The logical design of this Code Translator as well as additional considerations such as circuitry, special controls, and methods of operation are covered. Test runs indicate that nearly 100-percent compatibility between the two manufacturers' systems requirements has been achieved for the UNIVAC High-Speed Printer. From authors' summary

1187. Erismann, T., The analog-interpolator (in German), *ZAMP* **10**, 4, 339-347, July 1959.

Described is a method for transforming digital data into an analog representation. A stable parabolic interpolation is used, providing a smooth output. This method appears suitable for use in numerically controlled machine tools.

E. G. Newman, USA

1188. Cooper, A. J., and Hutcheon, I. C., An iterative analogue computor—to assist the study of the turbulent flame properties of elementary combustion chambers, Tiltman Langley Limited, Redhill Aerodrome, England, Tech. Note no. 3, 27 pp., Nov. 1959.

A transistorized iterative analog computer is described which has been built to solve up to 20 equations of the type

$$\tau_j = \frac{1}{L} \Sigma_k \, b_{jk} \, \psi \, (\tau_k)$$

where L is an eigenvalue, the b_{jk}'s are 400 coefficients and ψ is a linear or nonlinear relationship. It has been used to predict flame extinction conditions in simple combusiotn chambers.

From authors' summary

Analogies

(See also Revs. 1188, 1401, 1433, 1445, 1502, 1506, 1508, 1510)

1189. Sarkar, C. K., Effect of cut-offs in the design of weirs, a study by electrical analogy, *J. Technol., Calcutta* **4**, 1, 1-14, June 1959.

Paper presents some results obtained in determining through electrical analogy the distribution of the pressure head ratio under a weir foundation. Conducting graphite-impregnated paper is used for the investigations. Influence of vertical cut-offs placed at the ends of the structure as well as at two intermediate points is studied. On this basis, indications are given for obtaining the optimum position and depth of the cut-offs, corresponding to the minimum uplift pressure on the downstream part of the structure.

R.-J. Bally, Roumania

1190. Gzovskii, M. V., Modelling of tectonic fields of stresses and disruptions (in Russian), *Izv. Akad. Nauk SSSR Ser. Geofiz.* no. 6, 527-545, 1954; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5598.

The possibility of effecting the modelling described in the title is investigated; conditions of similarily are derived; results of the experiments are given. The author adopted the optical method for the determination of the magnitudes and directions of the maximum tangential and the principal normal stresses as best suited to the geological objects in view. As the result of modelling upheavals it was made quite clear, thanks to the optical method, that the maximum tangential stresses have low magnitudes in three regions —in the anticline and in the lower parts of the walls of the upheaval. It is assumed on the strength of these experiments that when the stresses are adequate the upheavals should intensify on the anticline from the top downward and on the walls from the bottom upward. Further modelling, in conformity with the principles of similarity, using as samples humid clay and abrasive powder, confirmed the author's assumptions.

B. L. Shneerson
Courtesy Referativnyi Zhurnal, USSR

Kinematics, Rigid Dynamics and Oscillations

(See also Revs. 1176, 1317, 1361, 1525, 1573, 1672, 1676, 1695)

Book—1191. Ziegler, H., Mechanics, Vol. 1: Statics of rigid bodies and elasticity [Mechanik, Band. 1, Statik der starren und flussigen Korper sowie Festigkeitslehre], 3rd revised ed., Basel, Switzerland, Birkhauser Verlag, 1960, 244 pp. Fr 28.50.

This text-book for students of the Engineering High School of Zurich is a third revised edition of "Mechanics" (Vol. 1); the earlier editions appeared in 1946 and 1948 and were written by Meissner and Ziegler.

Chapter I studies the statics of rigid systems and begins with composition and resolution of forces, equilibrium of current forces, polygon of forces. There are also some indications of graphical statics (the funicular polygon, the three-hinged arch, the Maxwell-Cremona diagrams) and analytical statics in the plane and in the space. At the end of the chapter some questions on friction and on flexible suspension cables are explained.

Chapter II deals with hydrostatics. It begins by explaining the notion of force-fields and then, given the equilibrium conditions of liquids, the equilibrium of floating (symmetrical) bodies is studied.

Chapter III deals with the strength of materials and studies the four fundamental cases of stressing (tension and compression, shearing forces, bending moment, torsion). Then, given some very simple examples of the bending of beams (external determination of forces acting on a section), the author considers internal tensions in the plane and in the space, shows Mohr's circles and studies the elastic and plastic strain and its energy. This permits resolving some statically indeterminate problems in beams, and the buckling of bars (Euler's formula).

At the end of the book Maxwell's and Castigliano's theorems are explained.

As is seen from this summary the volume gives a rapid view of the whole field of statics, with the exception of the principle of virtual displacements. G. Supino, Italy

Book—1192. Macmillan, W. D., Dynamics of rigid bodies, New York, Dover Publications, Inc., 1960, 478 pp. $2. (Paperbound)

This new Dover edition is an unabridged and unaltered republication of the first edition published by the McGraw-Hill Book Company, Inc. in 1936. Ed.

Book—1193. Routh, E. J., Dynamics of a system of rigid bodies, Elementary part, New York, Dover Publications, Inc., 1960, 443 pp. $2.35. (Paperbound)

This new Dover edition is an unabridged and unaltered republication of the author's last revised edition published by Macmillan and Company, Ltd. in 1905. Ed.

Book—1194. Malkin, I. G., Theory of stability of motion [Theorie der Stabilitat einer Bewegung], München, R. Oldenbourg, 1959, xiii + 402 pp. DM 47.

See AMR **13**(1960), Rev. 34.

Book—1195. Chandrasekhar, S., Principles of stellar dynamics, New York, Dover Publications, Inc., 1960, 313 pp. $2. (Paperbound)

This new Dover edition is an unabridged and unaltered republication of "Principles of stellar dynamics" originally published in 1942 by the University of Chicago Press. Ed.

Book—1196. Malkin, I. G., Some problems of the theory of nonlinear oscillations [Nekotorye zadachi teorii nelineinyh], Moscow, Gosud. Izdat. Tehn.-Teor. Lit., 1956, 492 pp. 16 r.

See AMR **10**(1957), Rev. 1002.

Book—1197. Cunningham, W. J., Introduction to nonlinear analysis, New York, McGraw-Hill Book Co., Inc., 1958, ix + 349 pp. $9.50.

This book is written by an electrical engineer and developed from courses given to electrical engineering students with, it seems, a rather average background in mathematics. In spite of the mathematically sounding title the author emphasizes that he wishes to teach the *use* of mathematical tools rather than to "devote time to the niceties of mathematics as such."

Within the framework of these self-imposed restrictions, the author does a creditable job. The result is a book useful as a textbook for graduate engineering courses, where there is certainly a need for books of this type.

The content may be seen from the chapter headings:

1. Introduction. 2. Numerical methods. 3. Graphical methods. 4. Equations with known exact solutions. 5. Analysis of singular points. 6. Analytical methods. 7. Forced oscillating systems. 8. Systems described by differential-difference equations. 9. Linear differential equations with varying coefficients. 10. Stability of nonlinear systems.

The treatment in most parts seems to be quite adequate, the weakest part being the one on forced oscillations in self-oscillatory systems, in particular on the phenomenon of entrainment of frequency.

Whereas the authors of many books on nonlinear oscillations restrict themselves to treating one method or one group of methods exclusively, the present book gives a survey on the many methods found useful in applications. The author keeps fairly close to his sources, which are well documented. This fact makes the book suitable also as a first guide to the pertinent literature.

K. Klotter, Germany

1198. Dornig, A., Transients in simple undamped oscillators under inertial disturbances, *ASME Trans.* **81** E (*J. Appl. Mech.*), 2, 217–223, June 1959.

When an elastically mounted machine contains an unbalanced rotating member the problem of accelerating or decelerating through the resonant condition is technically important. Author obtains a general solution for a one-degree-of-freedom system in integral form for arbitrary variation with time of the rotation of the eccentric mass. In particular the cases of uniform acceleration or deceleration through resonance are solved in closed form using Fresnel's integrals. It is shown that, for practical purposes, the maximum amplitude attained is, in both cases, a little more than three and one half times the steady-state amplitude at high speeds of rotation, multiplied by the quotient (resonant frequency/square root of acceleration or deceleration in revolutions per second per second).

P. C. Dunne, Brazil

1199. Zuk, W., and Majeed, M. A., A method of transforming concentrated surface forces into continuous surface forces, *J. Aero/Space Sci.* **27**, 6, p. 478 (Readers' Forum), June 1960.

1200. Novoselov, V. A., Motion of stabilized gyroscopic systems on a moving base, *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **23**, 5, 1375–1381, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Consider the motion of a stabilized gyroscopic system, assuming an arbitrary dependence on time of the motion of the base and of the mass of the gyroscopic system. Also the proper rotations of the gyroscopes are assumed to be nonstationary.

Usually, when investigating such systems a well-known method in the applied theory of gyroscopes is used. The equations of motion obtained by this method do not take into account the kinetic moments of the elements of suspension of the gyroscopic system nor those of the gimbals of the gyroscopes, nor the equatorial components of the kinetic moments of the rotors themselves, nor the kinetic moments of the motors. The paper gives a justification for author's method. As an illustrative example Sperry's ship gyrocompass "Mark II" is considered.

E. Leimanis, Canada

1201. Krementulo, V. V., Investigation of the stability of a gyroscope taking into account the dry friction on the axis of the inner Cardan ring (gimbal), *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **23**, 5, 1382–1386, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

An attempt is made to apply the direct method of Liapunov for the investigation of stability of certain motions of a gyroscope in a Cardan suspension, assuming dry friction on the axis of the gimbal.

E. Leimanis, Canada

Instrumentation and Automatic Control

(See also Revs. 1183, 1443, 1556, 1622)

1202. Schwartz, M. D., and Leondes, C. T., Extensions in synthesis techniques for linear systems, Parts 1 and 2, AFOSR TN 60–610 (Univ. California, Dept. Engng. Rep. 60–38), 55 pp., May 1960; AFOSR TN 60–609 (Univ. California, Dept. Engng. Rep. 60–39), 35 pp., May 1960.

These two reports seek to establish direct methods of synthesizing servos satisfying certain bounds on the error (or other performance criteria) when the input is given as a function of time. Actuating element is assumed to be an electric motor, and a method for selecting a motor of minimum power is developed.

The first design method is based on approximating the Laplace transform of the error by a quadratic with a constant numerator. The optimum parameters in this are determined, and hence a compensating network is derived, which makes the error a minimum according to certain criteria. The network may not be immediately realizable physically, in that it may have a transfer function with more zeros than poles. A realizable network is then obtained by adding poles of sufficiently large magnitude as not to affect the result significantly.

A second design technique makes use of the calculus of variations. The necessary variational techniques are derived ab initio. The object is to minimize the energy delivered to the load, but this cannot be done in an absolute sense since the minimum is reached when the output response is zero. The difficulty is circumvented by minimizing the time integral of a weighted sum of output power and square error, subject to certain alternative constraints on one of the integrands. The physical meaning of this is not brought out, and the reviewer is unable to assess its practical significance.

A third design technique is a variation of the first. By introducing a compensating network which is matched to the motor in some respects and arbitrary otherwise, the error is made approximately proportional to the rate of change of input. The error can then be minimized without leading to physically unrealizable networks.

The fourth design technique is concerned with the problem of avoiding saturation of the actuator motor.

The explanatory text is for the most part very badly worded, to the extent sometimes of disregarding both logic and grammar, and this makes the documents quite unnecessarily difficult to study (the actual mathematics are straightforward). The reviewer also objects to the practice of making statements alleged to be based "on physical reasoning" without stating what this reasoning is (e.g. a statement that "based on physical reasoning" the referred motor inertia is between one and ten times that of the load—actually it is often much higher).

The practical import of this work is very difficult to assess. The real question is whether it provides more accurate answers or requires less effort than cut and try methods, and it would probably require parallel design studies by skilled engineers to settle the matter.

R. Hadekel, England

1203. Woodrow, R. A., Data fitting with linear transfer functions, *J. Electronics Control* **6,** 5, 454-480, May 1959.

Paper investigates the fitting of passive linear systems with input data and output data. The impulse response function of the frequency response function is determined for the best data approximation. A cumbersome experimental investigation is thus unnecessary, if an approximate result is acceptable. Separation of closed loop systems from open ones is not necessary, both of them being investigated in the same manner. The solution of the problem leads to a set of integral equations which may be transformed into algebraic equations. As the number of these equations is lower than that of the unknowns, additional restraint conditions must be imposed. Various restraint conditions employed by some investigators without a previous justification are briefly presented. The best approximation through the minimization of the mean square error is justified by well-founded physical and mathematical considerations. This leads to an error uncorrelated with the input data $\phi_{x\varepsilon}(\tau) = 0$. By using Fourier transforms, frequency response functions are obtained with respect to spectral density functions. In the case in which the fitting is not exact, the coefficient η which indicates the degree of approximation is measured.

Paper examines simple closed and open loop systems and treats the general problem of a system with n inputs and one output. Four appendices are also included.

C. S. Pelecudi, Roumania

1204. Dunaev, V. I., Invariance principle in combined automatic control systems, *Automation and Remote Control* **20,** 5, 567-571, Feb. 1960. (Translation of *Avtomatika i Telemekanika* **20,** 5, 591-594, May 1959 by Instrument Society of America, Pittsburgh 22, Pa.)

The mathematical development treats a combined automatic control system; for example, a positional control system with two output motors and a differential to provide an algebraic summation of signals. One motor is controlled open loop and the other closed loop, and a zero dynamic error feedback control system is obtained by adjusting the combination in a straightforward mathematical manner. It would be of interest to determine how successful this technique is in practice or even in those analytical studies in which limiting is represented in two channels. Author does not treat this question.

Article apparently contains several misprints.

V. B. Haas, Jr., USA

1205. Kadymov, Ya. B., Methods of studying the stability of automatic control systems with distributed parameters, with allowance for losses, *Automation and Remote Control* **20,** 4, 500-503, Jan. 1960. (Translation of *Avtomatika i Telemekanika* **20,** 4, 525-527, Apr. 1959 by Instrument Society of America, Pittsburgh 22, Pa.)

Simple extension of Tsypkin's method [*Avtomatika i Telemekhanika* **9,** no. 3, 1948] to transfer function $\psi_1(s) \exp \tfrac{1}{2} yl - \psi_2(s) \exp -\tfrac{1}{2} yl$ where ψ_1, ψ_2 are polynomials and $\gamma^2 = cs^2 + bs$. Note is entirely theoretical, but analysis is spoiled by misstatement of convergence conditions for binomial series.

N. Ream, England

1206. Ito, M., Optimization of nonlinearly damped servomechanisms, *Mem. Fac. Technol., Tokyo Metrop. Univ.* no. 9, 655-664, 1959.

This paper discusses the response of a control system where the system error is governed by a second-order linear homogeneous differential equation with constant coefficients and the damping ratio is switched discontinuously between two (constant) values. The author seeks the optimum method of switching between these two values of the damping ratio so that the system error is reduced to zero in a minimum time with no overshoot. The switching curves in the phase plane turn out to be simply two straight lines through the origin. Of course, the time to reach the origin is infinite and no minimum time exists in the strict mathematical sense. The author does not explicitly state in what modified sense the response time is minimal.

N. J. Rose, USA

1207. Krasovskii, N. N., On the theory of optimum control, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **23,** 4, 899-919, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Problems of optimum control of systems described by differential equations

$$(dx/dt) = Ax + b\eta + \Sigma_{\alpha=1}^{n-1} e^{\alpha}\xi^{\alpha} \qquad [1]$$

where $x = \{x_1, \ldots, x_n\}$, $b = \{b_1, \ldots, b_n\}$, $e^{\alpha} = \{e_1{}^{\alpha}, \ldots, e_n{}^{\alpha}\}$ are n-dimensional vectors, A is an $n \times n$ matrix with constant elements, and η, ξ^{α} are scalar functions, are considered.

Given the initial conditions $x_0 = \{x_{10}, \ldots, x_{n0}\}$, it is required to find the functions η_0, $\xi_0{}^{\alpha}$ (optimum control) in such a way that the point $x(t) = x(x_0, t, \eta_0, \{\xi_0{}^{\alpha}\})$, moving along a trajectory of system [1], where $\eta = \eta_0$, $\xi^{\alpha} = \xi_0{}^{\alpha}$ reaches the origin of the coordinates $x = 0$ in the shortest possible time $t = T^0$ (T^0 being the optimum control time). It is assumed that the admissible functions $\eta(t)$, $\xi^{\alpha}(t)$ ($\alpha = 1, \ldots, n-1$) and the coefficients $e\beta^{\alpha}$ ($\alpha = 1, \ldots, n-1$; $\beta = 1, \ldots, n$) are subject to certain restrictions (Problems I to IV) on the controlling actions (force, current, stress), their energy or the impulses of the controlling quantities. Limit passages in the solutions are discussed which correspond to the passages from one type of restriction to another. On the basis of these limit passages, approximate methods are described for the computation of optimum trajectories and the construction of optimum systems. The method used is that of reduction of the Problems I to IV to the L-problem in specially selected functional spaces [see Krasovskii, *Avtomatika i Telemekhanika* **18,** no. 11, 1957], and Akhiezer and Krein, "On certain problems in the theory of moments," Article IV, p. 171, GONTI-NTVU, 1938 (Russian)].

E. Leimanis, Canada

1208. Pugachev, V. S., A method of solving the basic integral equation of statistical theory of optimum systems in finite form, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **23,** 1, 1-16, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N.Y.)

The problem of designing optimum linear filters for stochastic input signals has assumed great importance in modern servo theory since Wiener's fundamental investigations. The stationary problem has been almost completely solved; however this is not the case for the much more complicated nonstationary problem. The author considers nonstationary problems with a finite observation time interval and time-dependent statistical character of the input signals which in general entails time-dependent filter coefficients. The

filters are most easily characterized by their weighting function for the nonstationary problems. For optimum filters, a basic integral equation for the weighting function can be formulated with the known correlation function of the input signal as its kernel. The main filter design problem consists in solving this integral equation.

In this work the author summarizes and extends his earlier investigations and presents a quite general method for solving the integral equation. The method presupposes the knowledge of a differential equation transforming the actual input signal to white noise. Comparisons are made between this method and earlier known methods. The presentation is concise and therefore hard to read.

F. Y. Hjelte, Sweden

1209. Hsieh, H. C., and Leondes, C. T., Techniques for the optimum synthesis of multipole control systems with random processes as inputs, AFOSR TN 59-1132 (Univ. Calif., Dept. Engng.), 39 pp., Sept. 1959.

Paper concerns linear, multipole, and time-invariant systems whose inputs are random stationary processes. Authors aim at determining the physically realizable compensation system which minimizes the mean square error between the actual outputs and the desired outputs, the inputs being signals and noises. For the sake of clarity, the system with compensators placed in cascade with each fixed element of the plant is first treated, the error being independently minimized for each output.

Further, paper considers a group of multipole compensators, placed in cascade with the whole fixed plant, the total error being thus minimized for all output. The imposed conditions require that the weighting functions of the compensators satisfy a series of integral equations. The corresponding set of algebraic equations is obtained by Fourier transforms. Following a well-argued discussion on the placing of poles of the functions in the right and left hand sides of the equations, the components of the functions with poles in the upper half-plane are separated, a fact which leads to the determination of certain necessary coefficients by the identification of the residues. A simplification of calculus is to be noted when the transfer functions of the fixed elements are minimum phase.

Paper ends by presenting an illustrative example.

C. S. Pelecudi, Roumania

1210. Prousa, L., The stability of an extrapolation device, *Automation and Remote Control* **20,** 4, 493-496, Jan. 1960. (Translation of *Avtomatika i Telemekanika* **20,** 4, 518-520, Apr. 1959 by Instrument Society of America, Pittsburgh 22, Pa.)

Note treats by conventional z-transform techniques the stability of a sampled data feedback control loop with two integrations. Note apparently contains several misprints.

V. B. Haas, Jr., USA

1211. Piesch, J., Methods for analytical treatment of impulse-regulated control systems (in German), *Regelungstech.* **8,** 7, 238-244, July 1960.

The sequence of the impulses can be represented approximately by a series of Dirac impulses. Therefore, its Laplace transform F, i.e. the transfer function of the impulse-unit, is a series with the general term $f(nT) \exp(-snT)$ where T is the interval between the impulses. Introducing $z = \exp(sT)$ and $F^*(z) = \Sigma z^{-n} f(nT)$, author obtains the "$z$-transform" which can be used for dealing with the impulse-regulated system in the same way as the usual Laplace transform in the continuous case. It should be mentioned that the transform F^* has been defined and applied similarly by Cypkin in 1949 [*Avtomatika i Telemekanika* **10**].

W. Hahn, India

1212. Andrews, W. H., and Holleman, E. C., Experience with a three-axis side-located controller during a static and centrifuge simulation of the piloted launch of a manned multistage vehicle, NASA TN D-546, 26 pp., Nov. 1960.

An investigation was conducted to determine a human pilot's ability to control a multistage vehicle through the launch trajectory. The simulation was performed statically and dynamically by utilizing a human centrifuge. An interesting byproduct of the program was the three-axis side-located controller incorporated for pilot control inputs. This method of control proved to be acceptable for the successful completion of the tracking task during the simulation.

There was no apparent effect of acceleration on the mechanical operation of the controller, but the pilot's control feel deteriorated as his dexterity decreased at high levels of acceleration.

The application of control in a specific control mode was not difficult. However, coordination of more than one mode was difficult, and, in many instances, resulted in inadvertent control inputs. The acceptable control harmony at an acceleration level of 1 g became unacceptable at higher acceleration levels. Proper control-force harmony for a particular control task appears to be more critical for a three-axis controller than for conventional controllers. During simulations in which the pilot wore a pressure suit, the nature of the suit gloves further aggravated this condition.

From authors' summary

1213. Mahalanabis, A. K., Stabilization of contactor servos by using Coulomb friction, *J. Electronics Control* **8,** 4, 307-319, Apr. 1960.

1214. Summers, W. A., Starting an electric generating station—automatically, *Control Engng.* **7,** 9, 154-157, Sept. 1960.

1215. Magnus, K., Drift phenomena on oscillating Cardan suspended gyroscopes (in German), Advances in Aeronautical Sciences, Vol. I (Proc. of the First International Congress in the Aeronautical Sciences, Madrid, Sept. 8-13, 1958), Pergamon Press, 1959, 507-523.

Gyroscopic drift is caused by influence of gimbal friction and the distribution of the gyroscopic masses. Drift of this second kind appears even in a perfectly balanced gyro because of the presence of the gimbals. This is demonstrated in the investigation, and the drift is calculated for a frictionless two-gimbal gyro performing nutations. The analysis is also extended to a slightly unbalanced gyro. The reason for the drift appearing in the case of a balanced gyro is the nonlinear character of the governing differential equations.

Analysis is performed by using the second-order approximation of these equations and using first-order results to estimate second-order approximation of these equations and using first-order results to estimate second-order influence. The method is also applied to the case of a single-axis stable gyro platform. The last section estimates the gyro drift caused by gimbal motions when the vehicle carrying the gyroscope performs different flight modes (Dutch-roll motion and stationary turn).

F. Y. Hjelte, Sweden

Elasticity

(See also Revs. 1177, 1191, 1247, 1253, 1263, 1277, 1293, 1297, 1298, 1301, 1302, 1340, 1348, 1353, 1354, 1383, 1384, 1393, 1404, 1414, 1418)

Book—1216. Sneddon, I. N., and Hill, R., edited by, Progress in solid mechanics, Vol. 1, New York, Interscience Publishers, Inc., 1960, xii + 448 pp. $15.50.

Book contains eight chapters, each devoted to one particular branch of solid mechanics. Recent developments are reported and each chapter attempts to give a unified treatment of the subject material. In Chapter I, S. C. Hunter discusses viscoelastic waves. The theory is presented in terms of Boltzmann's hereditary integral representations of the stress-strain relations valid for linear viscoelasticity. Uniaxial wave propagation is considered in detail and it is shown that the Laplace transform is mostly of theoretical significance, while the Fourier transform leads to a description in terms of a complex modulus that can be determined directly in a vibration test. Experimental work on wave propagation is cited and the work of Kolsky on propagation of pulses of finite duration is shown to lend some support to the analytical results. Generalization of the equations of viscoelasticity into three dimensions is discussed briefly and attention is drawn to the need for experiments suitable for a discrimination between shear and dilatational behavior.

Chapter II gives a clear exposition by K. Marguerre of the application of so-called matrices of transmission to the vibration problem of beam structures. A thorough discussion of the problem of dynamic expansion of spherical cavities in ductile metals is given by H. G. Hopkins in Chapter III. After a rather detailed exposition of the physical problem and a discussion of the nature of possible discontinuities in physical quantities, both the solution for the quasi-static problem and the solution for the dynamic problem by means of Fourier integral transforms are considered for purely elastic material behavior. The subsequent treatment of the elastic-plastic analysis, meant to meet pressure conditions of greater practical significance, is based on Tresca's yield criterion and associated flow rule for plastic deformation. Small and large deformations are dealt with separately. It is shown how a crude approximation of actual material behavior already leads to considerable mathematical complexity. Nevertheless, some interesting results have been obtained and it is particularly noteworthy that the elastic-plastic boundary does not necessarily move at the plastic wave velocity.

A concise but admirably complete presentation of the general theorems in the mathematical theory of plasticity for "small deformations" is given by W. T. Koiter in Chapter IV. Here the terminology "small deformations" implies that the displacement gradients are infinitesimal. In the proofs of the uniqueness theorems, minimum principles, collapse and shakedown theorems elegant use is made of the virtual work equation together with the fundamental inequalities which are a consequence of Drucker's postulate for plastic deformation of a material that is stable in the small. With the exact Pochhammer-Chree solution for elastic wave propagation in a circular cylinder as a starting point, W. A. Green considers critically, in Chapter V, various approximate methods for longitudinal wave propagation in bars of other than circular cross section. It appears that the results obtained from approximate equations of motion tend to be more reliable than approximate solutions that satisfy the exact equations of motion and the boundary conditions to some degree of approximation. Paper concludes with a discussion of high-frequency solutions for a circular cylinder.

The dynamical theory of thermoelasticity is the subject of Chapter VI, written by P. Chadwick. The derivation of the basic equations is based upon the concept of local reversibility, though this is not mentioned explicitly. From a study of plane harmonic thermoelastic waves emerge the salient properties of thermoelastic processes in general. Particularly interesting is the result that the ceiling of attainable frequencies in a solid appears to be imposed not by the atomic structure but by thermoelastic damping. Further it is shown that for a perturbation of the deformation alone the problem is governed, to a very close approximation, by the equations of the dynamical theory of elasticity where the elastic constants should be given the adiabatic values. On the other hand when the deformation is due entirely to a heat-conduction process the displacement field in the solid is found by solving the equilibrium equations of the classical theory of elasticity, while the temperature distribution satisfies the heat-conduction equation to a very close approximation. In conclusion paper considers some thermoelastic boundary-value problems, giving prominence to the properties which they have in common. In Chapter VII B. A. Bilby gives an extensive account of recent attempts to incorporate into the continuum theory of the mechanics of deformable media the physical concept of crystal dislocations. By a formal limiting process whereby the density of dislocation lines tends to infinity while their strengths tend to zero the concept of the continuously dislocated crystal is introduced. It is shown that the tensor describing the dislocation density is essentially the torsion tensor, introduced by Cartan, of the appropriate non-Riemannian geometry. In terms of differential geometry a description is given of a state of deformation where the compatible shape deformation is resolved into a lattice deformation of a perfect crystal and a deformation determined by a continuous dislocation distribution. Paper contains some illustrative applications of the theory to states of stress and deformation. The dynamics of the deformation process are not yet covered by the theory. Further it did not become clear to the reviewer how the theory could be applied to a polycrystalline material.

Chapter VIII, written by R. Muki, deals with a generalization to the asymmetric case for a semi-infinite solid and a thick plate of Sneddon's method for the axisymmetric problems of the theory of elasticity. First it is shown that the solution of the Cauchy equations in cylindrical coordinates for the static case can be expressed in terms of a biharmonic function and a harmonic function. With respect to the angular coordinate these functions are expanded into trigonometric series, each term of which can be determined by means of Hankel transforms. Next, Hankel transforms are applied to a particular solution in cylindrical coordinates of the thermoelastic-displacement equations of equilibrium. The remainder of the paper consists of the solutions of special problems obtained by these general methods.

In reviewer's opinion this book is of great interest and value to workers in the field of solid mechanics.

J. F. Besseling, Holland

1217. Signorini, A., Questions of linear and semilinear elasticity (in Italian), *R. C. Mat. Appl., Univ. Roma* **18**, 1/2, 95-139, June/July 1959.

In these lectures the author recapitulates the fundamental relations of the theory of finite elastic deformations (a field of research where he has been active for many years) and shows how that theory can be used to obtain a rigorous background for the study of contact problems. In earlier papers [see, for instance: *Ann. Mat. Pura Appl.* (4) **30**, 1-72, Sects. 2 and 3] he introduced a perturbation process to provide a link between the solution of any hyperelastic problem and the solution of the corresponding problem within the classical linearized theory. Here he proves that, if unilateral surface constraints are allowed, the same process leads to *semilinear* conditions for the first approximation: the indefinite equations are the usual ones, but the boundary conditions are partly *ambiguous* (i.e. they may take either of two forms) and are partly expressed by inequalities. However, a theorem of unicity and the property of minimum for the potential energy remain valid within the semilinear theory.

There are many other details in this paper that will be appreciated by students of the theory of hyperelasticity: for instance, as far as possible, proofs are obtained without recourse to property of symmetry of the tensor (Part 1); it is shown (Part 2, Sect. 3) that the symmetric and antisymmetric parts of Kirchhoff's stress tensor are the derivatives of the elastic potential, if this is thought of as

a function of the linearized components of strain and of the vector of local rotation; a new example of incompatibility between the nonlinear and linearized theories is put forward (Part 3, Sects. 6 and 7).

G. Capriz, England

1218. Griger'ev, A. S., Two single problems of elastic equilibrium considering finite displacements (in Russian), *Inzbener. Sbornik Akad. Nauk SSSR* **26**, 85–86, 1958.

Author integrates the equations of elastic equilibrium of (1) a hollow sphere and (2) a cylindrical tube, under the action of constant radial pressure difference.

Reviewer asks why did author suppose the integrals valid for finite displacements while both the stress-strain relations and the measure of strain he uses are linear.

L. S. Rintel, Israel

1219. Zahorski, S., A form of the elastic potential for rubber-like materials (in English), *Arch. Mech. Stos.* **11**, 5, 613–618, 1959.

An expression for the elastic potential is obtained for (a) an isotropic compressible body and (b) an incompressible orthotropic body with curvilinear orthotropy, assuming that the principal directions of strain and orthotropy coincide. The results obtained constitute certain generalizations of known results (thus, for instance, in the case (a) one can obtain the Mooney material).

H. Zorski, Poland

1220. Williams, M. L., The strain analysis of solid propellant rocket grains, *J. Aero/Space Sci.* **27**, 8, 574–586, Aug. 1960.

Paper is mainly a long descriptive article on the title subject. The physical nature of the problem and some mathematical techniques for viscoelastic substances are discussed. Engineers and scientists wishing to learn something about the problems of burning grains of solid propellants could profitably read the article. Nothing new in the way of either analysis or physical principles is presented. Some data in the form of curves and charts as well as a select bibliography are provided.

W. H. Hoppmann, II, USA

1221. Romnov, Yu. I., On the effect of irradiation on stresses and small deformations in solids, *Soviet Phys.-Doklady* **4**, 1, 83–85, Aug. 1959. (Translation of *Dokladi Akad. Nauk SSSR (N.S.)* **124**, 3, 540–542, Jan./Feb. 1959 by Amer. Inst. Phys., Inc., New York, N.Y.)

Paper discusses connection between stresses and small deformations in a crystalline solid subject to radioactive irradiation (bombardment by heavy nuclear particles). Assuming isotropic character of the material, volume expansions have been determined due to (1) Frenkel defects (each nuclear fission liberates a unique amount of energy), (2) the formation of a second element in the nuclear reaction, and (3) the thermal effects. With volume expansion given, relations between small elastic deformations and stresses are developed.

A. Kezdi, Hungary

1222. Bramble, J. H., and Payne, L. E., A new decomposition formula in the theory of elasticity, AFOSR-TN 60-1129 (Inst. Fluid Dynam. Appl. Math., Univ. Maryland TN BN-218), 13 pp., Sept. 1960.

A new representation formula is given for the spherical components of displacement in the theory of elasticity. This formula may be used to reduce mixed boundary-value problems for the elastic sphere to standard problems in potential theory.

From authors' summary

1223. Aleksandrov, A. Ya., Some relations between the solutions of plane and axially symmetric problems in the theory of elasticity and the solution of axially symmetric problems by means of analytic functions, *Soviet Phys.-Doklady* **4**, 6, 1378–1382, May/June 1960. (Translation of *Dokladi Akad. Nauk SSSR (N.S.)* **129**, 4, 754–757, Nov./Dec. 1959 by Amer. Inst. Phys., Inc., New York, N. Y.)

Method used in author's previous paper now gives the relations between the solutions of plane and axially symmetric problems for a body bounded by a surface of arbitrary configuration. Solutions of the first and the second principal axially symmetric problems of theory of elasticity in the general case are given. The problem is reduced to the determination of two analytic functions.

Z. Olesiak, Poland

1224. Solyanik-Krassa, K. V., Solution of an axisymmetrical problem in elliptical coordinates (in Russian), *Trudi Leningrad Politekhn. In-ta* no. 196, 5–25, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5398.

An axisymmetrical stressed state in certain bodies of rotation is investigated—as ellipsoids of rotation and also in a body containing a closed ellipsoidal cavity. A method is adopted, based on the presentation of the solution of axisymmetrical problems in the theory of elasticity, in such a form that the stresses are expressed by the first derivatives of the stress function. Solutions are given for the following problems: compression (or tension) of an ellipsoid of rotation and also of a single-cavity hyperboloid of rotation by an axial force; the tensioning of a bar containing a small elliptical cavity; radial compression and deflection of a plate with an elliptical cavity. Numerical tables and graphs showing the distribution of the stresses are given.

Ya. S. Uflyand

Courtesy Referativnyi Zhurnal, USSR

1225. Kaufman, R. N., Green's functions and tensors in some boundary problems in the potential theory and the statical theory of elasticity for a layer and a semi-space with a spherical cavity (in Russian), *Inzb.-Fiz. Zb.* **1**, 7, 72–83, 1958; *Ref. Zb. Mekb.* no. 5, 1959, Rev. 5395.

By using the method of mirrored reflection Green's function is evolved for Dirichlet's problem in the theory of the harmonic potential, and Green's tensors for some particular boundary problems in the statical theory of elasticity for the regions indicated in the title. They are obtained in the form of infinite series, the coefficients for which are determined by the solution of some linear infinite systems of algebraical equations. It is shown that the method can be extended to embrace some other analogous regions.

V. D. Kupradze

Courtesy Referativnyi Zhurnal, USSR

1226. Abramyan, B. L., Some problems on the equilibrium of a round cylinder (in Russian), *Dokladi Akad. Nauk ArmSSR* **26**, 2, 65–72, 1958; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2875.

Several axially symmetrical problems are investigated: (1) the equilibrium of a continuous cylinder, radial displacements and tangential stresses having been assigned to its lateral surface, with stresses on its end; (2) the equilibrium of a slanting cylinder, axial displacements and tangential stresses having been assigned to its ends, with stresses on the lateral surfaces. The solution is made by means of series containing cylindrical functions.

V. K. Prokopov

Courtesy Referativnyi Zhurnal, USSR

1227. Imshenetskaya, E. F., On the formula for "discontinuity" for an integral met with in the theory of elasticity (in Russian), *Nauchn. Zap. L'vovsk. Politekhn. In-ta* no. 30, 15–23, 1955; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5396.

An investigation is carried out on the statical potential of a double layer in the theory of elasticity which plays a fundamental role in the theory of the first boundary problem (when displace-

ments are assigned on the boundary). The nucleus of this potential, as is known, [see V. D. Kupradze, "Boundary problems in the theory of oscillations and integral equations," Moscow, 1950, pp. 144 and 148] is formed from the fundamental solutions of the static equations in the theory of elasticity with the aid of a certain linear differential operator N, the so-called operator for pseudo-stresses (for the determination of N see ibid, pp. 125–129). In the paper under review the vector of stresses T is investigated, corresponding to the static field, expressed by the potential of the double layer, and it is shown that this vector undergoes disruption when the point being observed crosses the boundary of the surface being investigated; the value for the "discontinuity" is computed on the assumption of the existence of a multiple continuous differentiation of the compactness of the potential of the double layer. In concluding her paper the author, going astray, imagines that such a result contradicts one of the theorems of the abstractor, published in the cited book and designated there as the theorem analogous to the Lyapunov-Taubor theorem (see ibid, pp. 156–161); the "contradiction" in the author's view is explainable by a "calculation error" in the abstractor's proof.

Abstractor's remarks. Author's paper has been reviewed previously by N. A. Rostovtsev, who held the view that the Imshenetskaya result "corrects an error in one of the results" of V. D. Kupradze ["Boundary problems in the theory of oscillations and integral equations," Moscow, 1950, p. 157, theorem and formula (4.81)]. In the abstractor's work the study was of the N-operation on the potential of the double layer and proof is adduced for the theorem of continuity of that operation in the sense understood in the classical Lyapunov-Tauber theorem relative to the continuity of the normal derivative of the harmonic potential of the double layer; in Imshenetskaya's paper, however, the study concerns the T-operation (T being the stress operator) on the potential of the double layer, in consequence of which the continuity, admittedly, is disrupted in a similar manner to the disruption occurring for the tangential derivative of the ordinary potential of the double layer; it is also quite understandable that the Imshenetskaya result cannot be considered as an analog of the Lyapunov-Tauber theorem, the author's views notwithstanding. Besides, it can be demonstrated that if an investigation is made of the special potentials of the double layer, the nuclei of which are formed with the aid of operator T (and not of operator N) then the T-operation with similar potentials will be continuous on the boundary.

V. D. Kupradze
Courtesy Referativnyi Zhurnal, USSR

1228. Forsman, N. A., On stress concentrations of an extended bar of circular cross section (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 11, 25–30, 1958.

Approximate formulas for elastic, axially symmetrical problems of grooved and stepped bars under axial tension are developed. Method consists in applying statically self-equilibrating triangularly distributed shear stress to lateral surface of a cylinder with diameter equal to reduced diameter of grooved bar. Magnitude of maximum applied shear stress is obtained from known exact solution for bar with hyperbolic groove. Shear stress is applied over length determined by equilibrium requirement. Normal stress on lateral surface of cylinder is neglected. Solution for cylinder is expressed in established form of Fourier integral. This permits calculation of stress-concentration factor in terms of diameter of cylinder and depth and radius of curvature of groove. Procedure used for stepped bar is analagous.

B. S. Wilson, USA

1229. Piechocki, W., and Zorski, H., Thermoelastic problem for a wedge, *Bull. Acad. Polonaise Sci.* **7,** 10, 555–565, 1959.

Steady-state thermal stresses are analyzed in an infinite thin wedge with complete boundary restraint under a boundary temperature which is a general function of the distance from the vertex, followed by a temperature gradient across the boundary on the remainder of the wedge which is another general function of vertex distance. The wedge angle is a parameter. The temperature distribution is determined and applied to the static thermoelastic analysis of the stress field. Because of the steady-state plane temperature field, the stresses arise solely as a result of the boundary restraints.

G. Gerard, USA

1230. Stanisic, M. M., and McKinley, R. M., A note on thermal stresses in hollow cylinders (in English), *Ing.-Arch.* **27,** 4, 227–241, 1959.

1231. Bogomolov, A. D., Thermal stresses in a semi-space (in Russian), *Sb. Leningrad In-ta Inzh. Zh.-d. Transp.* no. 156, 167–171, 1958; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2897.

The procedure is given for the solution of the problem on the thermoelastic equilibrium of a semi-space by means of reflection on a circle of unitary radius and subsequent resolution of the values of the contour variables into Fourier series.

V. K. Prokopov
Courtesy Referativnyi Zhurnal, USSR

1232. Agarev, V. A., Umanskii, E. S., and Kvitka, A. L., Some questions in the solution of the thermal axisymmetrical problem in the theory of elasticity (in Russian), Questions in powder-form metallurgy and strength of materials, no. 5, *Kiev, Akad. Nauk USSR,* 1958, 134–159; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5400.

A general solution was obtained for the equations of equilibrium for the axisymmetrical problem in the theory of elasticity to meet the case where axial mass forces of constant intensity and centrifugal forces are acting. The linkage between this and known solutions is indicated. It is shown that in order to conform to the conditions of the joint nature of the deformation in the case of the axisymmetrical problem it is sufficient to satisfy two recorded equations. Some new general equations for the axisymmetrical thermoelastic problem are derived; these take into account the given mass forces and are convenient for use when the solution is accomplished with the aid of electric modelling. A detailed description is given for the procedure adopted for solving the problem for bodies of the type of a turbine rotor by means of a grid electrointegrator, with directions to methods enhancing the precision of the results.

A. Ya. Aleksandrov
Courtesy Referativnyi Zhurnal, USSR

1233. Newman, M., and Forray, M., Bending stresses due to temperature in hollow circular plates, Parts 1, 2, and 3, *J. Aerospace Sci.* **27,** 10, 792–793 (Readers' Forum), Oct. 1960; **27,** 11, 870–871 (Readers' Forum), Nov. 1960; **27,** 12, 951–952 (Readers' Forum), Dec. 1960.

1234. Ignaczak, J., Dynamic thermoelastic problem of a spherical cavity (in English), *Arch. Mech. Stos.* **11,** 4, 399–408, 1959.

The dynamic problem of elastic body with spherical cavity is considered in an accurate manner. The body is acted on by a time-variable heat source. The solution is composed of two parts: one in the form of a potential, and the other not expressible as a potential. This satisfies the boundary conditions (the vanishing of the stresses at the surface of the cavity), and the radiation conditions at infinity.

In particular cases the results obtained reduce to those obtained by other authors.

H. Zorski, Poland

1235. Smith, G. F., Further results on the strain-energy function for anisotropic elastic materials, Brown Univ., Div. Appl. Math. TR 7 (Contract DA-19-020-ORD-4725), 45 pp., August 1960.

1236. Kaliski, S., Fundamental solution for elastic and inelastic anisotropic bodies (in English), *Arch. Mech. Stos.* **11**, 5, 619–647, 1959.

The author obtains the basic solution for an elastic body of general rectilinear anisotropy, assuming that a perturbation propagates with finite velocity. The solution is obtained in the form of triple trigonometric series for any finite time interval $t < C$. The constant C is arbitrary and influences merely the convergence of the series obtained.

The proof of convergence is done for plane strain. Also the way of obtaining the solution for inelastic bodies is indicated. The paper contains a numerical example of a plane problem with certain simpifying assumptions concerning the elastic coefficients. Besides the series solution which is as yet the only solution suitable for numerical computation, the author gives, for an elastic body, a purely formal solution obtained by means of plane waves. This solution is expressed by a quadruple integral and is not convenient for actual computation.

It should be stressed that in view of the assumption, essential for the present paper, of finite propagation velocity we cannot pass to the static case. Z. Mossakowska, Poland

1237. Stanisic, M. M., and Sherwood, B. A., Torsion of an elliptical shaft with diametrically opposite flat sides, *J. Aero/Space Sci.* **27**, 6, 462–463 (Readers' Forum), June 1960.

The solutions for the torsional stress and torsional stiffness of elliptical shafts with flat sides which are presented in this paper have been obtained by means of the Galerkin technique. The solutions for both shear stress and torsional stiffness are given as functions of the ratio of semiaxes and of the ratio of flat-side distance with respect to major semiaxis. A comparison of the results obtained is made with those previously given in the literature. Close agreement in the solution is obtained. Numerical results of torsional stiffness and of maximal shear stresses for different shape ratios are tabulated.

From authors' summary

1238. Stanisic, M. M., Hauck, C. A., and Mathias, R. A., Concerning the torsion of prismatic bars having a hexagonal cross-section, *J. Aero/Space Sci.* **27**, 8, 631–633 (Readers' Forum), Aug. 1960.

The stress field, displacement field, and torsional rigidity which are presented for a prismatic bar of hexagonal cross section have been obtained by means of the Galerkin technique. For purposes of mathematical analysis of the problem only two terms of the stress function have been used. The convergence of the solution is demonstrated, and the resulting stress distribution between opposite corners and between opposite sides is tabulated. The analysis indicates that a maximum shear occurs just inside the boundary from the midpoint of a flat.

From authors' summary

1239. Simakov, F. F., Investigation of torsion systems (in Russian), Internal combustion motors, Moscow, Mashgiz, 1958, 58–92; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5502.

A "stress-deformation" diagram is examined for a system being subjected to torsional vibrations, with account being taken of internal friction produced between the particles of the material during deformation. The author brings forward a concept of the apparent rigidity of the shaft to withstand torsion, that is that type of rigidity which is obtained in computations of the resistance of materials when using the ordinary formulas, and of the actual rigidity of the shaft to withstand torsion, that is that type of rigidity which is obtained when dissipation of energy is taken into account. Further, author derives an expression for the torsional moment, expressed as the sum-total of the elastic moment and the moment of friction, an expression for the energy expended on the elastic twisting of the shaft and friction, and an expression for the actual modulus of elasticity of the second order for the system. A conclusion results to the effect that the modulus of elasticity of the second order and the rigidity of the shaft to withstand torsion are actually variables and depend on the stresses only in the region of small stresses, not exceeding 0.2 of the limit of elasticity of the material; while at higher stresses, not exceeding the limit of elasticity, both the modulus of elasticity as well as the rigidity of the shaft are practically independent of the deformation. When determining the frequency of the free vibrations the rigidity of the shaft to withstand torsion is taken to be subjected to stresses greater than 0.2 of the limit of elasticity, that is in a range of stresses where the modulus of elasticity for the material and the numerical value for the rigidity are practically constant.

A. F. Gurov
Courtesy Referativnyi Zhurnal, USSR

1240. Rvachev, V. L., The pressure on an elastic half-space of a stamp with a wedge-shaped planform, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **23**, 1, 229–233, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Guided by the solution to the contact problem when the contact region is the entire plane, the half-plane, and a semi-axis, the author introduces a stress function which satisfies the subject problem. W. C. Orthwein, USA

Book—1241. Dovnorovich, V. I., Spatial contact problems in the theory of elasticity [Prostranstvennye kontaktnye zadachi teorii uprugost], Minsk, Izdatel'stvo Belorusskolo Gosudarstvennolo Universiteta, 1959, 106 pp. 6 r. 65 k.

The determination of stresses and displacements in an elastic half-space loaded by rigid dies of various shapes is treated in this monograph. In all cases the die pressure acts in a direction normal to the face of the half-space. Attention is directed almost exclusively to the case when the displacements are given and it is necessary to find the corresponding distribution of pressures on the boundary plane.

The approach adopted in all problems is to consider the vector representing displacements in the elastic half-space and expand this vector in terms of Lamé functions so as to satisfy elastic relations. The pressure at an arbitrary point in the half-space is then determined from boundary conditions in terms of these same functions. The following types of rigid dies are considered: (a) elliptical contour, (b) general contour capable of being represented in terms of polynomial functions, (c) general contour capable of being represented in terms of the sum of even powers of a radial polar coordinate multiplied by either of the Cartesian coordinates. In each of these cases power series are used to represent the Lamé functions. No explanation is given for the use of these expansions when it is known that trigonometric expansions converge more rapidly.

The book is a useful addition to the literature of these specialized problems. It is rather well referenced.

W. A. Nash, USA

1242. Halling, J., The relationship between surface texture and rolling resistance of steel, *Brit. J. Appl. Phys.* **9**, 11, 421–428, Nov. 1958.

Experimental investigations into the relationship between surface texture of ground mild steel surfaces and the resistance to rolling of hard steel rollers are described. Results are discussed in relation to theories concerning interfacial slip, hysteresis and surface irregularities. Experimental observations show a decrease in coefficient of rolling resistance as surface finish improves and as the load increases. Although there is a definite dependence of

the rolling resistance on surface texture, other factors also contribute to the resistance, but the contributions are of a smaller order. C. F. Kettleborough, New Zealand

1243. Chankvetadze, G. G., A contact problem on an elastic semi-medium with a spherical cavity (in Russian), *Trudî Gruz. Politekhn. In-ta* no. 9 (57), 271–287, 1957; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5392.

An approximate solution is given for the problem on an elastic space with a spherical cavity into which is inserted a rigid sphere of the same radius, which is being subjected to a force applied perpendicularly to the boundary surface. The method used for the solution is that of conforming images, with the utilization of the resolution of the biharmonic spatial function of stresses into a series of homogeneous spherical biharmonic functions. The final formulas for the solution are complex and difficult of appraisal.

N. A. Rostovtsev

Courtesy Referativnyi Zhurnal, USSR

1244. Krasnen'kov, V. I., On the applicability of Hertz' theory of a three-dimensional contact problem (in Russian), *Izv. Vyssh. Uchebn. Zavedenii. Mashinostroenie* no. 1, 16–26, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5393.

For a die-stamping machine with a rectangular base, the length of which is considerably greater than the width, an approximate solution is found for the determination of the form of the surface of the machine which ensures an even distribution of the pressure over the length of the machine. The results appear to be of a qualitative character. It is established that the surface should bulge slightly, with a variable curvature for the transverse sections and that the initial contact should take place at a point. The distribution of pressure over the width coincides with that for an infinite strip. Having arrived at these conclusions, a deduction is put forward to the effect that Hertz' theory in regard to the elastic contact of infinitely long cylinders along the generatrix does not fully represent the conditions for the real contact of cylinders of finite length. N. A. Rostovtsev

Courtesy Referativnyi Zhurnal, USSR

Viscoelasticity

(See also Revs. 1190, 1216, 1252, 1351, 1357, 1368, 1376, 1423, 1427)

1245. Taira, S., Tanaka, K., and Ohji, K., A mechanism of deformation of metals at high temperatures with special reference to the tension test and the creep test, *Bull. JSME* **3**, 10, 228–234, May 1960.

Authors have set out previously [Proc. 7th Japan Nat. Congr. Appl. Mech. 1957 (1958) p. 31] a theory of deformation in which mechanical behavior is expressed in terms of strain rate, work hardening and recovery functions. These functions can in principle be determined by appropriate tests and used to predict results of other tests.

Present paper reports theoretical and experimental investigation of constant strain-rate tests (from 9 to 9×10^{-4} per cent per min) from which form of constant stress creep tests are predicted, all for a low carbon steel at 450°C. Creep tests are reported and compared with predictions. The reverse comparison between creep and strain rate tests is also made. Comparison shows very reasonable agreement over the relatively limited range of variables used. Authors' theory is claimed to be better than mechanical equation-of-state theory, in these circumstances.

C. E. Turner, England

1246. Kiziriya, G. V., Determination of the forces due to the shrinkage of concrete in redundant concrete constructions when taking into account the deformation caused by creep (in Russian), *Avtomob. Dorogi* no. 2, 22–23, 1958; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 3223.

The following relations are adopted from the time factor t, shrinkage of the concrete $S(t) = S_0(1 - e^{-\gamma t})$ and the deformation of creep $\varphi(t) = \varphi_m(1 - e^{-\beta t})$. Formulas are obtained for the stresses in concrete constructions with consideration for the displacement of the supports of the redundant constructions when the displacement conforms to the principle $\Delta(t) = \Delta_0(1 - e^{-\mu t})$.

G. M. Chuvikin

Courtesy Referativnyi Zhurnal, USSR

1247. Panarin, N. Ya., The influence of creep in concrete on the thermal stresses during fluctuations in temperature of the surrounding medium (in Russian), Sb. Nauchn. Trudî Leningrad Inzh.-Stroit. In-ta no. 26, 169–180, 1957; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5589.

The stresses referred to in the title are investigated in the light of the theory of creep given in the works of G. N. Maslov [*Izv. Vses. Nauk-i In-ta Gidrotekhn.* **23**, 1938] and of N. Kh. Arutyanin ["Some questions in the theory of creep," Moscow, Gostekhizdat, 1952]. M. A. Zadoyan

Courtesy Referativnyi Zhurnal, USSR

1248. Bykovskii, V. N., The resistance of materials with time, taking into consideration the statistical factors (in Russian), Moscow, Gosstroiizdat, 1958, 150 pp. + illus., 5 r; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5593.

The basic relations between the stresses and deformation, with the time element being taken into account (momentary elastic deformations, plastic deformations, and deformations due to elastic aftereffects), are enumerated. The material is investigated as an elasto-viscous nonhomogeneous dispersion medium; expressions are given for the coefficient of variability in relation to the dimensions of the test sample. Rheological schemes are presented for a nonstructural and a structural elasto-viscous medium. Basic differential equations are derived for elasto-viscous nonstructural and structural dispersion media in a mono-axial stressed state, the result of the integration of which is the obtaining of expressions for full deformation in functions of time and stresses. Statistical methods are applied to evaluate the irregular distribution of stresses caused by the nonhomogeneity of the stress field, and for the case of irregular distribution of stresses due to the nonhomogeneity of the medium.

A whole series of cases is investigated to find the statistical curves for the distribution of stresses. The magnitude is determined for the dispersion of stresses in cases of irregular stressed states. An investigation is made of the initial stage of plastic disruption, established by means of the magnitude of the effective limit of flow, in order to determine the limit of the carrying capacity of elasto-viscous dispersion media. A relation is obtained for the limit of flow of a structural system to time at a constant load, the minimum magnitude of which gives the value for the limit of protracted stability of the structural system. The linkage is found, in analogous fashion, of the limit of flow of a structural dispersion medium to the velocity of loading and the boundary state when loaded, which increases with the load increasing at a constant velocity. The relation between the stresses and the deformations and the determination of the limit for the carrying capacity in the mono-axial stressed state is transferred to the relation between the moment and the deflection in the case of bending.

The experimental investigation of wood is discussed. This commences with a study of the micro- and submicrostructure of clean wood and of the physico-mechanical properties of its constituent

parts. The coefficients of variability of wood are determined when the wood is subjected to axial loading; graphs are given to show the relation of the limits of stability and plastic flow to the velocity of loading and the dimensions of the test sample; the parameters are determined during the study of the process of deformation of the wood with time, for loading which is constant, or continuous or increasing step by step and for cases of compression and tension of the wood along the fibers. A study is carried out of the transition of the wood, under the action of a constant load, of compression and tension to the plastic stage of resistance and also of the process of its plastic disruption. Experimental studies are also undertaken on the transverse deflection of wooden joists, in the stage of permanent resistance and in the boundary state of disruption.

A brief investigation of the elasto-viscous and plastic properties of various amorphous and crystalline materials forms the conclusion to the publication. F. P. Belyankin

Courtesy Referativnyi Zhurnal, USSR

1249. Gusev, V. N., Calculations of flexible coverings viewed as elasto-viscous-plastic systems (in Russian), *Trudi Mosk. Avtomob.-dor. In-ta no.* 23, 118–126, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5617.

A road surface is investigated as a viscoelastic body which can be described by means of a model composed of four elements comprising linear elastic and elastic elements and a nonlinear viscous one. The viscosity of the last is determined from the equation $\eta_1 = A/\sigma - E_{equiv}\,\varepsilon$, where σ is the stress, ε the deformation, A and E_{equiv} parameters. The author thinks it possible to use the equation for creep with σ = const for the computation of the road covering (or soil) deformation under the action of motor traffic, disregarding the changes of stress in space and time. It is possible to determine the service life of the covering or, should the velocity of the motion of the vehicular traffic be known, the number of trips of the motor cars up to the road going out of commission, provided the maximum critical deformation is assigned. An equation is obtained for the road covering corresponding to the model proposed in the paper. Nomograms are drawn enabling the service life of the covering to be determined in relation to the loading and the numerical values of the viscous-elastic constants of the material. A method is proposed for the experimental determination in field conditions of the viscoelastic parameters of the covering.

N. I. Malinin

Courtesy Referativnyi Zhurnal, USSR

1250. Hoff, N. J., A survey of the theories of creep buckling, AFOSR TN 50-382 (Stanford Univ., Dept. Aero. Engng. no. 80), 60 pp., June 1958.

In first part of paper author illustrates static and dynamic approaches to solution of creep buckling problem considering simplified model of a column. Linearly viscoelastic column, nonlinearly viscous and viscoelastic columns and strain-hardening column are treated for this model. In second part brief review of papers on creep buckling of columns is given. A few simple approximate methods suggested by various authors are discussed in some detail. Creep buckling of plates and shells is dealt with in two brief chapters. Paper gives good discussion of physical aspects of creep buckling problem. F. J. Plantema, Holland

1251. Hoff, N. J., Mechanics applied to creep testing, AFOSR TN 59-39 (Stanford Univ., Dept. Aero. Engng. no. 83) 80 pp., Dec. 1958.

This lecture before SESA gives general review of problems connected with creep phenomena, illustrated by various simple examples. Successive chapters deal with: (1) Empirical creep laws for primary and secondary creep at constant stress and temperature; variable stress; variable temperature; multi-axial states of stress. (2) Causes of scatter in creep tests, due to inaccuracies of stress or temperature, stress or temperature fluctuations and changes in geometry of test specimen. (3) Stress distribution in presence of secondary creep; elastic and plastic analogs. (4) Interaction between elastic deformation and creep, leading to behavior similar to primary creep. (5) Instability phenomena under tensile or compressive loading; creep rupture in tension, necking, creep buckling.

Avoidance of complicated mathematics and emphasis on physical aspects make paper very suitable for obtaining insight into problems dealt with. Chapter on creep buckling can be considered as abstract of author's separate paper SUDAER 80 (see preceding review). F. J. Plantema, Holland

Plasticity

(See also Revs. 1190, 1216, 1283, 1361, 1364, 1391)

Book—1252. Likhtman, V. I., Rebinder, P. A., and Karpenko, G. V., Influence of a surface-active medium on the processes of deformation of metals (in Russian), Moscow, Izd-vo Akad. Nauk SSSR, 1954, 208 pp. + illus. 10 r 40 k; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 3323.

The results are published of the work done by the department of dispersion systems of the Institute of physical chemistry of the Acad. of Sci. of the USSR over the past 10 to 15 years. The phenomena produced by the adsorption interaction of the deforming metal with the surrounding medium containing surface-active substances are described; the lowering of the limit of flow, the increase in the velocity of creep, the lowering of the fatigue resistance and the electrocapillary effect of relieving the deformation of metals are discussed. I. M. Gryaznov

Courtesy Referativnyi Zhurnal, USSR

1253. Kroner, E., and Seeger, A., Nonlinear elasticity theory of dislocations and residual stress (in German), *Arch. Rational Mech. Anal.* **3**, 2, 97–119, Apr. 1959.

The non-Riemannian geometry of dislocations introduced by Kondo and independently by Bilby, Bullough and Smith is extended to describe a nonlinear theory of dislocations and residual stress.

One can conceive residual stress as a constraint which forbids the body to leave Euclidean space. If this constraint could be suddenly removed, the body would relax in a certain non-Euclidean space. In other words to the body as a whole there corresponds a natural state whose geometry is non-Euclidean. This idea is developed mathematically and applied, by means of stress functions, to yield second-order approximations to the stresses in certain problems of rotational and translational dislocation. One great advantage of the method used by the authors is that the stresses are expressed as functions of the coordinates of the particles in the deformed state. L. M. Milne-Thomson, USA

1254. Hodge, P. G., Jr., Yield conditions for rotationally symmetric shells under axisymmetric loading, *ASME Trans.* **82** E (*J. Appl. Mech.*), 2, 323–331, June 1960.

The yield condition for a rotationally symmetric shell may be represented as a surface in a four-dimensional stress space. The exact yield surface according to the Tresca yield condition is compared with various approximations. A new approximation is suggested which combines the advantages of mathematical simplicity and reasonable accuracy. The theory is illustrated with reference to a spherical cap under uniform pressure.

From author's summary by H. Fernandez Long, Argentina

1255. Hill, R., Some basic principles in the mechanics of solids without a natural time, *J. Mech. Phys. Solids* **7,** 3, 209–225, June 1959.

This paper contains a number of physical remarks and new terms concerning classical elasticity, plasticity, hypoelasticity, and related theories. Results concerning uniqueness and stability are asserted in considerable generality. While a good many equations are written down, the hypotheses and definitions are not stated in sufficient detail for this reviewer to follow the arguments.

Reviewer's note: Since this paper was published, an adequate general theory of elastic stability has been constructed by Coleman and Noll, *Arch. Rational Mech Anal.* **4**(1959/1960), 97–128, 1959. The author's assertion on p. 220 that anisotropic elastic materials are necessarily hypoelastic has been shown to be false by Bernstein, *Arch. Rational Mech. Anal.* **6,** (in press).

C. Truesdell, Italy

1256. Sokolovskii, V. V., Longitudinal displacement of plastic mass between non-circular cylinders, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **23,** 4, 1043–1053, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N.Y.)

Author considers rough noncircular cylinders and develops a method of reducing solution of various problems having nonlinear law of deformation to solution of same problems employing linear law. Fundamental equations and transformation of equations are presented. As an example author considers field of shear stresses and longitudinal displacements for case when inner and outer cross-section contours of cylinders are confocal ellipses. Author carries out numerical example and concludes that technique as presented could be used to investigate other problems of longitudinal displacement of plastic mass between noncircular cylinders.

V. A. Valey, USA

1257. Balan, S., Rautu, S., and Petcu, V., A new method for studying the behaviour of structures in the elastoplastic range—Chromoplasticity (in English), *Rev. Mech. Appl.* **4,** 1, 97–111, 1959.

This paper deals with "chromoplasticity," a new method for studying the behavior of constructions in the elastoplastic range. The principal propetty of the chromoplastic material employed by the authors, a vinyl polychloride, is a change of color at the points in which the stress reaches the yield limit.

The use of models of such a material allows the possibility of following visually the appearance of zones of plastic deformations, and the formation of plastic hinges, up to the loading capacity of the system. The determination of the order of appearance of the plastic hinges is quite simple, especially in the case of more complex structures in two or three dimensions.

Some of the tests performed by the authors are shown with pictures indicating the practical arrangement of the work.

Reviewer believes this method, because of the simplification it means, will be very useful in the application of "plasticity" to the study of structural systems of greater complexity.

J. L. Delpini, Argentina

1258. Marukhno, G. P., A method of solution for variational problems in the theory of plasticity (in Russian), Report of the 16th Scientific Conference of the Professional Lecturing Staff of the Leningrad Engng.-Constr. Inst., Leningrad, 1958, 609–611; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2998.

It is shown that when use is made of the principle of the minimum of work due to deformation (with variation in the stresses) for the solution of some problems in the theory of ideal plasticity it proves convenient for the selection of approximating functions to utilize the relation for the modulus of plastic shear (which is part of Hanky's equation)

$$\psi = \frac{T}{T_y\sqrt{3}}$$

where T is the limit of flow during tension, T_y is the intensity of the tangential stresses in an elastic solution, G is the modulus of elastic shear. This gives a possibility of obtaining a linear system relative to the sought parameters for the equations.

G. S. Shapiro

Courtesy Referativnyi Zhurnal, USSR

1259. Dmitriev, A. D., Plastic deformation of infinite plates on an elastic base during prolonged action of axisymmetrical shearing stresses (in Russian), *Trudi Saratovsk. Avtomob.-Dor. In-ta* **15,** 1, 31–41, 1957; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 3003.

This is an engineering attempt to solve the stated problem on the assumptions that the reaction of the elastic base of the plate at any point is proportional to the deflection of the plate at the same point; the dominating influence on the form of the deflection is exercised by the tangential and not the normal stresses; the tangential stresses are proportional to the ductility of the material. The second assumption is contrary to the conditions of the theory of deflection of beams and plates. Tangential stresses remote from the points of application of concentrated forces play a secondary role in comparison with normal stresses. The possibility of using elementary methods of investigation close to the points of application of forces as claimed by the author requires substantiation. The author holds the view that the tangential stresses are evenly distributed over the thickness of the plates. This contradicts the requirement of zero tangential stresses on the surface of the plate.

G. S. Shapiro

Courtesy Referativnyi Zhurnal, USSR

1260. Tsygankov, E. M., Determination of the magnitude of the deformations when strips are being plastically deflected (in Russian), *Trudi Gor'kovsk. Politekhn. In-ta* **13,** 7, 35–44, 1958; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 3004.

An investigation is carried out of the plastic deflection of strips in the cold and in the hot state. It is shown that when the ratio of the radius of deflection R to the thickness of the strip b is less than 5 the calculations for the magnitude of the longitudinal deformation should be made starting from the functions for curved rods. Results of experiments are given which show that for the case $R/b < 5$ an appreciable diminution in the thickness and area of the transverse section of the strip is observable. The influence is experimentally investigated of the width of the strip on the magnitude of the transverse deformation during deflection. It is shown that the magnitude of the transverse deformation decreases as the width of the strip increases; in this case the transverse deformation is only observable at the edges at distances equal to twice the thickness of the strip.

Yu. A. Rakovshchik

Courtesy Referativnyi Zhurnal, USSR

1261. Shapiro, G. S., On a rigid-plastic annular plate under impulsive load, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **23,** 1, 234–241, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N.Y.)

1262. Blaha, F., Langenecker, B., and Oelschlagel, D., Plastic behavior of metals under the influence of sound (in German), *Z. Metallk.* **51,** 11, 636–638, Nov. 1960.

1263. Gatewood, B. E., The problem of strain accumulation under thermal cycling, *J. Aero/Space Sci.* **27,** 6, 461–462 (Readers' Forum), June 1960.

Rods, Beams and Strings

(See also Revs. 1172, 1216, 1228, 1238, 1255, 1302, 1308, 1317, 1381, 1393, 1395, 1401, 1405, 1664, 1668)

1264. Dabrowski, R., Simultaneous bending and torsion of thin-walled bars (in German), *Stahlbau* **29**, 4, 104–111, Apr. 1960.

In design of crane girders, combined bending and torsion frequently governs stresses and deformations. Problem was rigorously solved by Pettersson (1949 and 1952) for mono-symmetrical and bisymmetrical cross sections. Author uses Ritz method to obtain approximate solution of problem, which is generally within a few per cent of exact solution. Reviewer considers that application of author's equations is more time-consuming than using diagrams by Pettersson. G. G. Meyerhof, Canada

1265. Broude, B. M., The strength of slightly distorted and eccentrically loaded double-I beams (in Russian), Calculations for extensive constructions, no. 4, Moscow, Gosstroiizdat, 1958, 5–35; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5544.

An approximate solution is given for the problem on the loss of strength of the second order in a thin-walled double-I beam with hinged fastening and free deplanation of the beam's ends. The problem is solved by the method of small excitations. The following assumptions are made: (1) when determining the deflection-torsion deformations of the beam the material is taken to be elastic with no limit; (2) the critical state of the beam is identified by the loss of strength in its compressed flange, which is viewed as a compressed-deflected bar; (3) the influence of the tangential stresses on the strength is ignored; (4) the conditions for the loss of strength are taken in the form of

$$\sup [\sigma + \chi \Delta\sigma] = \sigma_T$$

Here the symbol sup designates the upper boundary of the function, σ is the stress along the axis of the flange at the moment of loss of strength, $\Delta\sigma$ is the supplementary stress due to deflection and torsion in the edge fiber of the flange, σ_T is the limit of yield of the beam's material. Stresses σ and $\Delta\sigma$ are determined on the assumption of the elastic work of the material. The coefficient χ, less than unity, takes into account the presence of plastic deformation. A table is furnished for the values of χ found for cases of eccentric compression. These values for χ hold good for other types of loading of the beam.

The determination of the elastic deformations is carried out by means of the resolution by natural functions of the corresponding system of homogeneous equations. To do so V. Z. Vlasov's equations ["Thin-walled elastic beams," Moscow, Gosstroiizdat, 1940] are employed. The natural functions are determined by the Bubnov-Galerkin method. It should be noted that in the general case the natural functions are not orthogonal due to the fact that the equation for solving the problem contains different steps of the load's parameter. A beam with initial distortion is examined for a case of pure deflection and for the general case of symmetrical loading. The eccentric loading of a beam during deflection is investigated for the cases: (1) when two concentrated moments are acting on the ends of the beam; (2) when the concentrated force is at the center of the span (in various cases of application of the force by the height); (3) when the load is evenly distributed (also for cases when the load is in different positions). Tables are given, to assist the carrying out of practical computations, for the critical stresses and calculation coefficients, based on the mean value of χ taken to be equal to $1/\sqrt{2}$, and the limit of yield $\sigma_T = 2400$ kg/cm^2 in relation to the parameters of the beam and the magnitude of the eccentricity (the indicator of the initial distortion). A comparison of the theoretical data with the experimental shows that they were mutually close to each other when the eccentricities were small. V. A. Mar'in

Courtesy Referativnyi Zhurnal, USSR

1266. Shaikevich, V. D., The matrix method for the calculations of regular beam systems (in Russian), Calculations for large constructions, no. 4, Moscow, Gosstroiizdat, 1958, 145–181; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5648.

In his previous work [see Sb. nauchn. Prak. Dnepropetr. Inzh.-Bud. In-ta, no. 3, 111–130, 1957] the author had developed a method of matrix focal relations. In the present paper he transposes this method to the calculations for regular frames by using an infinite system of equations. It is not pointed out that regular multistoried and multispan frames are very rarely used. The matrices, which appear to be a generalization of focal relations, are then used for the calculations of matrices of influence (that is, reversible matrices). The proposal is made to utilize the method of iteration, without imposing any limits for the calculation of the values of the unknown, but this is not always possible. The study utilizes the known fact that from the canonical system of equations, recorded for regular frames, it is possible to derive three-celled matrices which are actually generalizations of Jacobi's numerical three-termed matrices. Jacobi's matrices are ordinarily not regular in the mathematical sense of the word, consequently the method of iteration is not applicable for the solution of the corresponding systems of equations for the general case.

The reading of this paper has been made difficult because of the unusual terminology adopted by the author, which is unknown both in the theory of matrices and in structural mechanics.

L. K. Narets

Courtesy Referativnyi Zhurnal, USSR

1267. Vyaz'menskii, S. P., The general theory for nonlinear problems of small elastic deformations of three-dimensional beams (in Russian), Report on the 16th Scientific Conference of the Professional Lecturing Staff of the Leningrad Engng.-Constr. Inst., Leningrad, 1958, 546–550; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5658.

Paper describes an attempt to bring up-to-date the classical theory of small elastic deformations of large beams under the action of arbitrary loads by taking into fuller consideration the influence of torsion, as follows: (1) conservation in the equations for the derivatives of the transposition components and their derivatives on the angle of torsion and its derivatives, (2) consideration of the constricted nature of unevenly applied torsion, (3) effect of the increase in the torsional moment due to the influence of normal stresses in fibers which have been forced out of position in relation to their axis during twisting of the beam. It is noted that selection of the factors of nonlinearity taken into account limits the field of application of the results. Special cases have not been investigated. The whole of the exposition is synoptical.

Ya. B. L'vin

Courtesy Referativnyi Zhurnal, USSR

1268. Pestel, E., Application of delta matrices to nonhomogeneous problems (in German), *Ing.-Arch.* **27**, 4, 250–254, 1959.

Further discussion of transmission matrix method as developed by Fuhrke AMR **9**(1956), Rev. 1008 and others. Author treats illustrative beam problem in detail. See also contribution by Marguerre in "Progress in solid mechanics," Vol. I (1960), edited by Sneddon & Hill, for extended discussion and bibliography.

T. P. Mitchell, USA

1269. Rappoport, R. M., The statics of thin-walled rods consisting of branches connected by plates (in Russian), Calculations for large structures, Gosstroiizdat no. 4, 239–282, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5681.

It is assumed, as the result of investigations of compound beams, that these beams can be replaced by an equivalent continuous layer provided there is a sufficient number of discrete links available for disposition. The paper records an attempt to evaluate the error due to this substitution, special attention being given to the torsion of the beam, consisting of branches joined together rigidly by plates. It is shown that the elementary solution, investigating the panel of the composite beam as a thin-walled beam with an unchanging contour for the transverse section, results in significantly reduced transpositions and excludes the possibility of taking into account the difference between the free and constricted torsion.

A. V. Dyatlov

Courtesy Referativnyi Zhurnal, USSR

1270. Yashin, V. F., Computations for beams lying on an elastic foundation (in Russian), *Trudi Mosk. In-ta Inzb. Zb.-d. Transp.* no. 94, 150–166, 1957; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5666.

The reactive pressure on the beam is adopted in the form of a stepped three-term equation, with an expression for the sag in the form of a simplified formula for the influence of the whole of the pressure; a known integral condition of joint deformation (the coincidence of the areas of the graphs of the deflection of the beam and settling of the foundation) is recorded; the coefficients of the stepped three-term equation are given in the form of prepared formulas. Examples and tables are furnished with the object of simplifying the computations.

N. K. Snitko

Courtesy Referativnyi Zhurnal, USSR

1271. Neville, A. M., and Lord, E., Some factors in the shear strength of reinforced concrete beams, *Struct. Engr.* **38,** 7, 213–223, July 1960.

The different types of shear failure of reinforced-concrete rectangular and T-beams are described in order to show that the shear strength depends on factors additional to those recognized by the design formulas. Results of experiments on the influence on shear strength of the size of the compression zone of the beam and of hooks on plain and deformed bars are presented. Tests on rectangular beams with varying amounts of compression reinforcement suggest that this does not affect the shear capacity of the beam.

From authors' summary by K. Angervo, Finland

1272. Csonka, P., Bending forces in block steps of supported stairs, loaded along a single stair-groove (in English), *Acta Techn., Acad. Sci. Hungaricae, Budapest* **25,** 3/4, 321–334, 1959.

Author analyzes stresses and deflections in a flight of stairs in which one step is subjected to load distributed along the edge of the step. Each step is considered to behave as a beam simply supported at the ends and joined to the steps above and below it by "hinges" which transmit shear but not moment from one step to another. Distribution of load among the steps is found to depend on the ratio of torsional stiffness to bending stiffness for each step. Numerical examples are included.

J. W. Clark, USA

1273. Petre, A., Thin-walled curved rods (in Russian), *Rev. Mech. Appl.* **4,** 3, 521–527, 1959.

Author considers a curved rod whose wall is curved both in the cross section and along its length. It is assumed that the shear strain in the cross section is equal to zero. The expression of the displacement along the tangent to the rod axis is a deduced function of the displacement component in the cross section.

Further, Hooke's law is admitted for the normal stress having the direction of the tangent to the rod axis and its expression is obtained with respect to the displacements in the cross section. The tangential stress in the wall is also obtained. It is to be noticed that this stress is not equal to zero although according to Hooke's law for the tangential stress and to the previously accepted hypothesis it must be equal to zero.

Finally, the expression of forces and moments in the cross section is deduced with respect to the displacements in this section.

M. M. Misicu, Roumania

1274. Bassali, W. A., The torsion of elastic cylinders with regular curvilinear cross sections, *J. Math. Phys.* **38,** 4, 232–245, Jan. 1960.

The torsion problem of the cylinder the cross section of which is mapped conformally onto the unit circle by the mapping function $z = \omega(\zeta) = c\zeta/(1 + m\zeta^n)$ has been solved by the author by substituting the value of $\omega(\zeta)$ in the known Muskhelisvili's formulas of torsion function and torsional rigidity quoted by Sokolnikoff ["Mathematical theory of elasticity," New York]. The author finds the values of the shearing stresses and also the maximum and minimum values of the tangential stress on the boundary for negative values of m.

D. N. Mitra, India

1275. Pestel, E., Application of transfer matrices to the torsion of a box section (in German), *ZAMM* **38,** 11/12, 465–466, Nov./Dec. 1958.

Let the load system of a rectangular, doubly symmetric and uniform box section consist of the torque in the web sheets of the hull and of tensional forces at the longitudinal ribs. The states of stress and strain are then characterized by six quantities: the main and additional shear flows, the longitudinal forces at the stiffening ribs, the angle of twist per unit length, the shear strain and the warping of the cross section. Two transfer matrices are presented, one of them regarding two cross sections with no transverse sheet between them, the other regarding the cross sections on both sides of a transverse sheet.

P. Laasonen, Finland

1276. Shimogo, T., On the beam under random loading, *Bull. JSME* **3,** 9, 60–65, Feb. 1960.

In this paper several statistical properties of the response of a pin-ended beam having viscous resistance to changes in curvature are calculated for a particular forcing function which is random both timewise and spanwise.

The paper is an interesting and worthwhile attempt aimed at the eventual statistical design of an airfoil, and the forcing function used is the random lift on an airfoil moving through three-dimensional turbulence; however, the choice of hinged end conditions and the assumption that the scale of turbulence is much smaller than the length of span make the paper seem somewhat artificial to the reviewer.

P. G. Kirmser, USA

Plates, Shells and Membranes

(See also Revs. 1254, 1304, 1305, 1306, 1307, 1353, 1354, 1366, 1393, 1413, 1662)

1277. Salerno, V. L., and Goldberg, M. A., Effect of shear deformations on the bending of rectangular plates, *ASME Trans.* **82** E *(J. Appl. Mech.),* 1, 54–58, Mar. 1960.

Reissner theory is applied to a simply supported plate with constant load. The boundary conditions approximate the situation in a riveted sheet-stringer combination where the edges are held against tangential rotation. Reviewer thinks it could be foreseen that the deviation from the classical theory is small. Authors' boundary conditions reintroduce the Bernoulli hypothesis (left out by Reissner) along all edges of the plate. But Reissner's theory is known to differ from the classical one only near the edges of the

plate in accordance with Saint Venant's principle. [See Manfred Schäfer, *ZAMM* **32**, 6, 161–171, June 1952; AMR **6**(1953), Rev. 809].
H. Schaefer, Germany

1278. Kirillov, V. S., The theory of calculations for oblique-angled plates, supported along the contour (in Russian), *Trudi Mosk. Avtomob.-Dor. In-ta* no. 21, 111–127, 1957; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5448.

A solution was found expressed in ordinary series for the problem of the deflection of a thin elastic plate freely supported along the contour, the plate being in the form of a parallelogram. An investigation is made on the action of an evenly distributed load over the whole of the plate and over an area having the form of a rectangle, two sides of which are parallel to the edges of the plate, and also over the parallelogram with sides parallel to the edges. The solution is carried out in rectangular coordinates, which is in contrast to the method used in a number of published works, where the deflection of freely supported oblique-angled plates is examined in terms of oblique-angled coordinates, which are more convenient to use in their case.

A. Ya. Aleksandrov
Courtesy Referativnyi Zhurnal, USSR

1279. Adadurov, R. A., An axially symmetric state of stress in a thin annular plate, *Soviet Phys.-Doklady* **4**, 1, 221–224, Aug. 1959. (Translation of *Doklady Akad. Nauk SSSR (N.S.)* **124**, 5, 1005–1008, Jan./Feb. 1959 by Amer. Inst. Phys., Inc., New York, N. Y.).

Author investigates the state of stress in a round annular plate which is attached at its internal and external radii, r and R, to perfectly rigid rings and loaded by moments M applied to these rings in their planes. The plate is assumed to be thin and incapable of carrying compressive stresses. Under the indicated loading, creases form in the plate and in the direction of a crease the tensile stresses $\sigma_c = \sigma_{max}$ are principal stresses. In the direction perpendicular to a crease, normal stresses are $\sigma_{min} = 0$. Shear stresses in directions parallel and perpendicular to a crease are zero. Such a state of stress has been called uniaxial. This state of stress in the plate is likewise axially symmetric. The exact solution of the linear formulation of this problem is given.

From author's summary by V. Bogunovic, Yugoslavia

1280. Reggini, H. C., Solution of the problem of elastic thin plates (in Spanish), Instituto de Investigaciones Cientificas y Tecnicas de las Fuerzas Armadas Publicaciones no. 8, 42–59, June 1959.

The plate equations and boundary conditions are written in terms of the two bending moments. As an illustration, the corresponding difference equations are solved numerically for two cases concerning a rectangular plate with one free edge; a good approximation is attained even with a large mesh size.

P. Cicala, Italy

1281. Leyko, J., The problem of stress distribution in a compressed rectangular plate subject to simultaneous action of transverse load (in Polish), *Arch. Budown Maszyn* **6**, 2, 259–277, 1959.

Problem formulated in the title is reduced to a solution of Kármán's equations for large deflections of plates. The deflection w is represented approximately by a Ritz expression, and the equation for the stress function is solved explicitly. The remaining unknown parameters are found by applying Galerkin's method to the second Kármán equation. An example involving rectangular freely supported plate is solved in detail using one parameter representation of w. To improve the accuracy of the coefficient of reduced stress (ratio of mean value of stress to maximum stress in the middle plane) the same problem is analyzed using perturbation method.

J. Nowinski, USA

1282. Lukasiewicz, S., Approximate method of calculating deflections and stresses in triangular cantilever plates with variable rigidity (in Polish), *Arch. Budown Maszyn* **6**, 2, 307–324, 1959.

The problem formulated in the title is reduced to solution of Euler-Lagrange equations for the corresponding minimum potential energy equation. The deflection of the plate is approximated by $f_0(t) + \vartheta f_1(t)$, and more accurately by $f_0(t) + \vartheta f_1(t) + \frac{1}{2}(3\vartheta^2 - 1) f_2(t)$ where f_i are arbitrary functions of position t along the span, and ϑ is coordinate measured in transverse direction. Numerical examples are given for transverse cross sections of the plate resembling ellipse and rhombus, with power variation of rigidity along the span. Stresses are compared with experimental data (hardly visible on the graph).

J. Nowinski, USA

1283. Pirvu, A., Bending of plates (in Roumanian), *Studii Si Cercetari Stint. Mat.* **10**, 1, 153–156, 1959.

Author considers an elastic-plastic thin circular plate of constant thickness, loaded by bending moment uniformly distributed at its boundary. Material is assumed to be homogeneous, isotropic, obeys Hooke's law until it yields and exhibits no strain hardening. Hencky-Mises yield criterion is applicable. Radial displacements are given by the same formula in both elastic and plastic ranges, resulting from the assumption that the normal lines remain straight during deformation. Axial displacements in plastic ranges are derived from the condition of incompressibility of material.

It appears that some printing errors, not affecting the results, should be corrected.

C. Hoschl, Czechoslovakia

1284. Bulakh, G. D., Deformation of a circular concrete plate (in Russian), *Nauchn. Trudi Odessk. In-ta Inzh. Morsk. Flota* no. 13, 13–21, 1957; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 3219.

An approximate method is proposed for taking account of the distortion stresses when the upper surface of an isolated circular plate is cooled. By selection of a conditional computation scheme the problem merges with the utilization of known solutions: on the collapse of an absolutely stiff sphere in an elastic semi-space and on the deformation of a circular plate, freely supported along its perimeter loaded with an axisymmetrical load. The condition of deformation, governing the use of known solutions enables the maximum intensity of the reaction on the base and the collapse in depth of the center of the plate to be found. The example of numerical calculations given contains several errors.

A. L. Kvitka
Courtesy Referativnyi Zhurnal, USR

1285. Shaishmelashvili, V. N., Calculation for a collapsed plate by the method of finite differences (in Russian), *Trudi Naucbn. Korr. In-ta Stroit. Dela. Akad. Nauk GruzSSR* **2**, 61–89, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5425.

A thin-walled sloping shell of variable curvature is examined. The problem is solved on the basis of V. Z. Vlasov's momentary technical theory of shells ["The general theory of shells," Moscow-Leningrad, Gostekhteorizdat, 1949] by means of strips, that is by merging differential equations having special derivatives with ordinary differentially-differing equations. To solve the latter, different variational methods are adopted: Fourier series, Vecu's method, the method of finite differences. A detailed record is available for the different boundary conditions for the strips. Data are furnished of a numerical example of calculations for a square collapsed plate.

P. N. Varvak
Courtesy Referativnyi Zhurnal, USSR

1286. Pogorelov, Yu. D., Investigation of a plane stressed state of plates having tapered outlines (in Russian), *Trudi Khar'kovsk. Inzh.-Stroit. In-ta* no. 7, 21–31, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5444.

An approximate method of calculation by means of variational computation and the use of a simplified calculation procedure is proposed for plates of tapered outline. According to this procedure the surface of the plate is divided into a number of separate bands, each band having the hypothesis of plane sections applied to it. The sought components of stress, situated in the longitudinal cuttings of the band, are determined from the condition of minimum potential energy of elastic deformation of the system. The differential equations due to Euler which emanate from such condition are obtained with divided unknowns, which eases very considerably the solution of the problem for plates with many steps. The theoretically obtained approximate formulas were verified by experimental methods of photoelasticity on optically active material, a phenol aldehyde plastic. K. V. Zakharov

Courtesy Referativnyi Zhurnal, USSR

1287. Karavanov, V. F., Equations for slanting three-layer shells with a light filler during finite displacements (in Russian), *Izv. Vyssh. Uchebn. Zavedenii, Aviats. Tekhn.* no. 1, 69–77, 1958; *Ref. Zh. Mekh.* No. 5, 1959, Rev. 5420.

The equations cited in the title are given. The equations were obtained after making the following basic assumptions: the longitudinal forces and moments are taken up by the external layers, while the filler reacts only to the shear and the transverse deformations of tension-compression (a light filler). A system is obtained for the slanting shell from fourteen equations, analogous to Reissner's equations for a three-layer plate [E. J. Reissner, *J. Aero. Sci.* **15**, no. 7, 1948]. Besides, the natural deflection stiffness of the outer layers is disregarded in the study and the filler is considered to be connected to their middle surfaces. When the transverse deformation of the filler is not taken into account the system merges with one having two equations; E. I. Grigolyuk [*Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 1, 77–84, 1957] also obtained analogous equations by disregarding the transverse deformation of the filler, while for the case of a cylindrical shell where account was taken of the transverse deformations of the filler the abstractor also achieved these results [*Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 3, 142–144, 1958]. L. M. Kurshin

Courtesy Referativnyi Zhurnal, USSR

1288. Tikhonova, L. G., A cylindrical shell compressed along its center by two forces (in Russian), Mekhanika (MVTU **50**), Moscow, Oborongiz, 1956, 245–260; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2908.

This is an investigation of the action of two concentrated forces on a cylindrical tube, of finite and infinite length; the forces are coaxial but reversed to each other in direction, they are applied along the center normal to the surface of the shell, the edges of the tube of finite length being assumed to be freely supported. The problem regarding the equilibrium of the tube of finite length is solved by means of the Navier method, while the solution for the infinitely long shell is sought in the form of a Fourier series for the periphery and a Fourier integral for the generatrix. Three variants of the equations for the solution are compared: (1) the so-called precise equations obtained as the outcome of the Kirchhoff-Lavie hypothesis, (2) equations for the determination of the stressed state with a large indicator of variability, (3) equations for the determination of the stressed state of the type of a degenerated boundary effect of the cylindrical shell. The numerical results obtained (for shells of relative thickness $h/R = 0.01$, $h/R = 0.02$) confirm the general situation of the theory of cylindrical shells as regards the extent of applicability of the above variants for calculation: variant (2) is unsuitable for the calculation of long shells, variant (3) gives incorrectly the stressed condition in the case of short shells, and of long shells as well, in the vicinity of the point of application of the concentrated force. The data for the experimental work are given and also the numerical results of the calculation in the form of eight curves.

N. A. Alumyae

Courtesy Referativnyi Zhurnal, USSR

1289. Immerman, A. G., The verification of the strength of round cylindrical shells when subjected to the joint action of longitudinal and transverse loads (in Russian), Calculations for extensive constructions, no. 4, Moscow, Gosstroiizdat, 1958, 525–530; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5432.

The upper limit of strength of round cylindrical shells is determined when they are being subjected simultaneously to even axial loads p and transverse loads q. The determination follows the solution of the problem of the strength of such shells by S. V. Aleksandrovskii [Calculations for large constructions, no. 3, Moscow, Gos. izd-vo Lit. po Str-vu i Arkhitekt., 1955, 453–492]. A formula is given for the critical axial load, which in the case of large transverse loads is converted into the form

$$p = 0.92 \frac{R}{l} \frac{E\mu^{3/2}}{k/\mu} \qquad \left(\mu = \frac{t}{R}, \quad k = \frac{a}{p}\right)$$

where E is Young's modulus, R the radius of the mean surface, t the thickness, l the length of the shell. The limits to be imposed on the derived formulas: $l/R > 2.5$ and $k/\mu > 0.2$ are fixed on the basis of the experimental data. In the original title the word "shells" was omitted. P. I. Zheluev

Courtesy Referativnyi Zhurnal, USSR

1290. Troisvallets, R., and Montel, R., Tests concerning the collapse of smooth metal pipes embedded in concrete and subjected to external pressure (in French), *Houille Blanche* **15**, 4, 364–390, June 1960.

The problem under consideration is that of the resistance of unreinforced linings to external pressures. The authors describe the large program of tests they conducted on sections of concrete-embedded 4,100-mm diameter pipe with varying wall thicknesses and with shape defects and clearances between the plating and the concrete. The article deals with the composition of the sections that were tested, with the methods and equipment used for the tests and measurements (measurement of initial shape followed by measurements of stress and strain during the tests), and gives the test results. Analysis of the results brings out the special pattern to which the shape defects conform as well as the effect they have on the localization of the collapse wave and on the limiting pressure. They also confirm the effect of the clearances between the concrete and the plating. These results and comparison with various methods of computation lead to the conclusion that the shape defect factor, which has previously been neglected, should be allowed for when determining the pressure under which linings will collapse. The method of using a template to measure shape defects gives coherent results.

From authors' summary

1291. Ikhvanin, I. I., The strength of a conical shell of circular section when pressure is being evenly exerted along the generatrices (in Russian), *Dopovidi Akad. Nauk URSR* no. 3, 267–271, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5427.

An investigation is made of the symmetrical and the asymmetrical forms of stable equilibrium when a conical shell is being compressed along the generatrix. The setting for the problem is linear. The problems being examined are not novel and have been investigated previously in the works of I. Ya. Shtaerman [Sb. Trudí

Kievsk. Aviats. In-ta, no. 1, 1936] and of Kh. M. Mushtari [*Prikl. Mat. Mekh.* **7**, no. 7, 1943]. A. V. Sachenkov

Courtesy Referativnyi Zhurnal, USSR

1292. Novinstki, Yu., Thin-walled beams of double curvature and large conicity (in Russian), *Byul. Pol'skoi Akad. Nauk Otd.* (4) **4**, 2, 95–98, 1956; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5428.

A method is proposed for the calculation of thin shells of rotation viewed as thin-walled beams with consideration for the deplanation of the section and disregard of the contour's deformations. The principle of similarity along the meridian is adopted for normal and shearing forces. A. N. Elpat'evskii

Courtesy Referativnyi Zhurnal, USSR

1293. Neuber, H., Developable shell-beams (in German), *ZAMM* **40**, 1/3, 22–38, Jan./Mar. 1960.

Consider a metal beam-like shell, the ends of which are rigidly attached to other parts of a structure; restraining forces and couples at one end are related to those at the other end through "flux" of stress in the surface which is assumed free from applied loading laterally. Now, if the shell surface is developable, a membrane solution can easily be found, treating the shell surface as a Riemannian V_2 immersed in Euclidean 3-space. Parameters u, v, on V_2 not necessarily orthogonal, but, after development of surface, any differential element is such that orthogonality is preserved; one of the principal curvatures is zero. In the developed surface, the longitudinal direct force S and the shear force T are physical components of a tensor L. The hoop stress is proved to be zero. Components of restraining forces and couples are expressed as line integrals of L; results are then expressed in terms of two stress functions, the determination of which constitutes the general solution for discussion of which a variational method is used. Both continuously closed, and open, cross sections are considered. Example of circular cone is worked in detail; formulas for direct and shear stresses are obtained, but author has not pointed out that these could, in certain obvious circumstances, produce infinite stresses.

Paper assumes knowledge of elementary Riemannian geometry. It is easy to read; argument is logical and clear. Reviewer's only criticism is that some symbols could have been defined in more detail. Author is content to refer to theory expounded in a much earlier paper [*ZAMM* **29**, pp. 97–108, 142–146, 1949].

F. A. Gerard, Canada

1294. Shaishmelashvili, V. N., Calculations for slanting shells by the strip method (in Russian), *Soobshch. Akad. Nauk GruzSSR* **18**, 2, 197–204, 1957; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2916.

The equations for the title problem are converted into differentially-differing equations by substituting for their particular derivatives in one direction their approximate values—i.e. central finitely differential relations. The solution of the obtained equations is sought in the form of generalized Fourier series by girder-type fundamental functions satisfying the boundary conditions of the problem at the ends of the strips. In order to find the unknown coefficients of the series the variational method is applied which gives an approximate solution of the problem. An examination is made of the different boundary conditions at the ends of the strips and on the border strips; series are derived as described and also finitely differential relations.

O. D. Oniashvili

Courtesy Referativnyi Zhurnal, USSR

1295. Klingbeil, E., Theory of shells of rotation from the standpoint of numerical analysis (in German), *Ing.-Arch.* **27**, 4, 242–249, 1959.

The equations of linear isotropically elastic thin shells of revolution of variable thickness subjected to static loads with axial symmetry are put in the form of four coupled first-order ordinary differential equations with arc length along the meridian as the independent variable. This is considered to be a useful form for machine computation although there is no discussion of the numerical problems; e.g., stability of a marching technique when applied to an equilibrium problem. S. H. Crandall, USA

1296. Obolashvili, E. I., A problem on the moment-free equilibrium of a composite shell (in Russian), *Soobshch. Akad. Nauk GruzSSR* **19**, 6, 649–652, 1957; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5426.

Two shells are examined; they are connected along the boundary; the mean surfaces of the shells appear to be a spherical segment and a segment of a paraboloid of rotation respectively. The force components are expressed with the aid of analytical functions and with consideration for the fact that the forces acting along the common boundary should be continuous. The Riemann-Gilbert problem for a circle is obtained for expressing the analytical functions, the solution being obtained by Schwartz' formula.

R. K. Ryayamet

Courtesy Referativnyi Zhurnal, USSR

1297. Tsurkov, I. S., Elastic equilibrium of rectangular panel-type shallow shells undergoing finite deflections (in Russian), *Inzhener. Sbornik Akad. Nauk SSSR* **26**, 49–53, 1958.

Attention is given to the large deflection of a rectangular elastic plate, simply-supported and free to move inward at its edges, under transverse static load. Straightforward analysis is given of the von Kármán equations. The final objective is to obtain a simple approximate solution of these equations, and some illustrative numerical results are presented. The references given are solely to the author's previous work. H. G. Hopkins, England

1298. Knowles, J. K., and Reissner, E., On stress-strain relations and strain-energy expressions in the theory of thin elastic shells, *ASME Trans.* **82** E (*J. Appl. Mech.*), 1, 104–106, Mar. 1960.

The stress-strain relations of Flügge and Byrne for thin elastic shells are inverted to express strain quantities, and therewith the strain energy, in terms of stress resultants and couples. In this form, and upon omission of terms which are small of order h^2/R^2, the stress-strain relations and the strain-energy expression are shown to be simply related to corresponding results of Trefftz. The strain-energy formula of Trefftz is generalized to arbitrary orthogonal middle surface coordinates.

From authors' summary by H. Schaefer, Germany

1299. Rudiger, D., The Ritz and Trefftz methods in shell theory (in German), *ZAMM* **40**, 1/3, 114–123, Jan./Mar. 1960.

Using tensor notation, paper exhibits general form of equations for energy methods of Ritz, Trefftz and Galerkin for shells. Paper is mainly of theoretical interest.

G. W. Housner, USA

1300. Sankaranarayanan, R., On the dynamics of plastic spherical shells, AFOSR TN 60-1165 (Polyt. Inst. Brooklyn, Dept. Aero. Engng. Appl. Mech., PIBAL Rep. no. 580), 20 pp., Sept. 1960.

This paper discusses the behavior of complete spherical shells and simply supported spherical caps under impulsive loading by uniform external pressure. The pressures are assumed to be greater than the static collapse pressures and to act for a short period of time. The displacements of the shells are presented for particular values of the parameters of the problem.

From author's summary

1301. Akasaka, T., and Takagishi, T., The shear modulus of foil honeycomb sandwich structures, *Trans. Japan Soc. Aero. Space Sci.* **2**, 3, 83–90. 1959.

Approximate formulas are derived for the effective shear modulus of sandwich elements having a foil honeycomb core. Analysis assumes that: (1) cores are capable of resisting only shearing stresses in a plane normal to the longitudinal axis of the cells, (2) cell walls do not buckle, (3) shearing stresses do not vary parallel to the longitudinal axis of the cells, and (4) Poisson's ratio of the facings is zero. Since the assumed core construction of double thickness walls parallel to the sandwich facings and single thickness oblique walls produces marked orthotropic elastic properties, formulas are derived for the following conditions: (1) shear force applied parallel to the facings, (2) shear force applied normal to the facings, and (3) shear force applied in an arbitrary direction. By comparing the effective shear modulus of a core without facings to that of a sandwich having facings of infinite flexural rigidity, authors show that for a honeycomb having hexagonal cells the facings will contribute a maximum of 10% to the shear rigidity.

Analytical results are compared with limited experimental data for cores having hexagonal cells. For a shear force applied parallel to the facings, the authors' analysis is in close agreement with the experimental data. However, for the case of a shear force applied normal to the facings, the theoretical analysis predicts an effective shear modulus considerably larger than that found experimentally. Authors attribute this lack of agreement to initial deflections and buckling of the cell walls.

W. K. Rey, USA

Buckling

(See also Revs. 1250, 1251, 1393)

1302. Muller, P. H., and Kummer, H., The practical determination of nonlinear eigenvalues and application of the process to a stability investigation (in German), *ZAMM* **40**, 1/3, 136–143, Jan./Mar. 1960.

Authors prove that eigenvalues of certain integral equations are approached as limit of eigenvalues of approximating matrix equations, and apply method to elastic stability problem previously considered by Prandtl.

C. E. Pearson, USA

1303. Beedle, L. S., and Tall, L., Basic column strength, *Proc. Amer. Soc. Civ. Engrs.* **86**, ST 7 (*J. Struct. Div.*), 139–173, July 1960.

This paper provides a review of research pertaining to the influence of residual stress on the strength of centrally loaded structural steel columns. Experimental work at Lehigh University is summarized and column strength is related to the tangent modulus, not of the individual test coupon, but rather the tangent modulus of the average stress-strain curve of a short stub section of the entire column cross section.

The deleterious effect of residual stress is most apparent for effective slenderness ratios represented by $50 < L/r < 100$. Greater relative strength reduction occurs for buckling about the weak axis than for buckling about the strong axis. Primary attention is given to rolled wide flange shapes, but built-up welded or riveted sections, low-alloy steels, and variation in yield point are other topics that are briefly discussed.

The importance of this research work arises from the fact that a rational evaluation of structural steel column strength is now possible without any consideration of accidental curvature or end eccentricity.

B. G. Johnston, USA

1304. Harris, L. A., and Auelmann, R. R., Stability of flat, simply supported corrugated-core sandwich plates under combined loads, *J. Aero/Space Sci.* **27**, 7, 525–534, July 1960.

Buckling curves are presented for the general instability of flat, simply supported, corrugated core, rectangular sandwich plates under longitudinal compression and bending, transverse compression and bending, and shear. The stability criterion is formulated by means of the Rayleigh-Ritz energy method in which the deflection surface due to shear is assumed to be of the same form as the deflection surface due to bending and twisting. The solution of the eigenvalue problem for the buckling coefficients was performed by a matrix iteration on an IBM 704 computer.

From authors' summary by B. E. Gatewood, USA

1305. Radkowski, P. P., Buckling of thin single- and multilayer conical and cylindrical shells with rotationally symmetric stresses, Proc. Third U.S. Nat. Congr. Appl. Mech., June 1958; Amer. Soc. Mech. Engrs. 1958, 443–449.

Initial buckling load of thin elastic multilayer shell under rotationally symmetric loads is computed by means of Ritz method. The torsional case is not considered. The surface whose elements do not experience any strains under bending is used as the reference surface. Each layer is supposed to be of constant thickness and isotropic. The usual assumptions, such as preserving of the normal elements, are used.

The solution is divided in two groups: the first includes cylinders and truncated cones with both base radii nearly equal; the second includes other cases.

For the first group, besides given approximate formulas, the results are plotted in graphs; for the second group the results are given only graphically. Author suggests a simple method also for cases where a part of the shell is subjected to tension. Reviewer believes that the resulting curves may be interpreted as an upper bound of the real buckling loads. Some experimental data show that for lower bound the theory of finite deflections should be used, eventually with regard to initial imperfections of the shell.

I. Hlavacek, Czechoslovakia

1306. Hill, D. W., Buckling of a thin circular cylindrical shell heated along an axial strip, AFOSR TN 59–1250 (Stanford Univ., Dept. Aero. Engng. SUDAER no. 88), 91 pp., Dec. 1959.

In the introduction author states that the thin cylinder is assumed to be simply supported on its ends so that the radial and circumferential displacements there are zero, but the axial displacement and end rotations are not restricted. Under these end conditions the axial stresses due to heating a narrow strip will vary in axial direction and will be accompanied by shear stresses which necessarily lead to severe bending of the shell. Therefore it is not surprising that the author after some preliminary experiments was compelled to perform his tests on cylinders either with the ends restrained or with two diaphragms at each end so as to form closed cells.

On the basis of linear shell theory, Ritz's method is applied to obtain a theoretical solution of the thermal stress and thermal buckling problem by means of Fourier series modified by a cleverly chosen shape function leading to rapid convergence.

Courant's maximum-minimum principle is used to establish that the buckling stress under uniform axial compression is a lower bound for all other cases of axial loading, provided that the buckling stress is taken as the maximum compressive stress on the cross section. Results for buckling tests on eleven cylinders are presented and shown to be in reasonable agreement with the theoretical predictions. In reviewer's opinion more detailed experimental data would have enhanced the value of the paper for future reference.

J. F. Besseling, Holland

1307. Lotkin, O. I., Stability of thin-walled shells of rectangular cross section on elastic supports (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 11, 113–116, 1958.

The Bubnov-Galerkin method is applied to case involving uniformly distributed transverse load and eccentrically applied end load. A table of numerical values of quantities appearing in characteristic equation is given. B. S. Wilson, USA

Vibrations of Solids

(See also Revs. 1268, 1276, 1393, 1541, 1694)

1308. Wieckowski, J., The phenomenon of resonance in semi-infinite elastic beams (In Polish), *Rozprawy Inz.* **7**, 4, 417–442, 1959.

Author considers steady-state forced transverse vibration of straight semi-infinite beams with constant cross section under the action of a concentrated force or moment varying in a harmonic manner and applied to any point of the beam. Homogeneous initial conditions and a number of types of end conditions are assumed. The damping is disregarded. That portion of the beam between the end and the attachment point of the force is called the "resonator," the remaining part being called the "sink."

The object is to investigate the phenomenon of "resonance." According to the author's definitions, at a point of coordinate x we are concerned with the phenomenon of deflection, slope, or longitudinal "resonance," if the corresponding quantity reaches its maximum as a function of the frequency.

The "resonance" in the "sink" or in the "resonator" takes place when the multiplicity factor in the "sink" or in the "resonator" reaches its maximum as a function of excitation frequency. Flexural vibration is considered, taking the rotational inertia into consideration. The phenomenon of "dynamic obstacle" is discussed. This develops when the mechanical energy does not pass through the point of excitation, a certain amount of energy being thus captured within the resonator.

In the author's opinion the "dynamic obstacle" is connected with the problem of quantization of the phenomenon of elastic stability and energy levels, described in the works by J. Naleszkiewicz. K. Piszczek, Poland

1309. Adem, J., Matrix differential systems with a parameter in the boundary conditions and related vibration problems, *Quart. Appl. Math.* **17**, 2, 165–171, July 1959.

Certain physical systems such as multiple-degree-of-freedom vibrating systems may be described by a column matrix of functions rather than by a single function. If matrix notation is used to solve these problems, the analysis leads to a matrix eigenvalue problem which is a generalization of the Sturm-Liouville problem but can be treated along the same general lines as the scalar problem. In the first part of this paper, certain vibration problems that give rise to matrix eigenvalue formulations are discussed. The vibration of a string with concentrated mass points and different densities between the mass points is discussed by the matrix method. The paper concludes with a discussion of the general matrix differential system, and the general properties of the eigenvalues and eigenvectors of the system are deduced.

L. A. Pipes, USA

1310. Payne, L. E., and Weinberger, H. F., Some isoperimetric inequalities for membrane frequencies and torsional rigidity, AFOSR TN 60-292 (Inst. Fluid Dynam. Appl. Math., Univ. Maryland, TN BN-194), 11 pp., Feb. 1960.

Using the isoperimetric inequality, authors show that the highest fundamental frequency of a membrane of given area and perimeter, fixed on an outer boundary and free along any inner holes, is attained when the membrane is of annular form.

A similar result is obtained if the membrane is elastically supported on the outer boundary. Employing the resemblance between the principal frequency of a membrane and the torsional rigidity of a beam, the method is also applied to this last case, a similar result being obtained. A. L. Petre, Roumania

1311. Gladwell, G. M. L., and Bishop, R. E. D., Interior receptances of beams: Part 1, Tables of interior receptances; Part 2, Derivation of the interior receptances, *J. Mech. Engng. Sci., Lond.* **2**, 1, 1–15, Mar. 1960.

Vibration analysis of systems with finite number of degrees of freedom has been attacked by the authors as well as by others using the receptance concept. See, for instance, AMR **8**(1955), Rev. 1280; **10**(1957), Revs. 396, 635, 1336, 2424; **12**(1959), Rev. 5998; **13**(1960), Rev. 4523. Dynamic behavior of a structure and computation of its natural frequencies can be carried out directly and systematically when knowing the receptances (mechanical admittances or dynamic flexibilities) of the individual members. This paper—an extension of previous ones—explains a method for finding the response at an interior point of a beam to the excitation of any other interior point (or, as the authors call it, the "interior receptance"). Tables of results are presented for uniform beams. H. C. Reggini, Argentina

1312. Black, H. F., Application of Macaulay's method to vibrating beams, *J. Mech. Engng. Sci., Lond.* **2**, 1, 69–76, Mar. 1960.

Complete non-series solutions for steady-state vibrations of uniform-section beams with all possible combinations of various supports are given, using a generalization of the Macaulay notation commonly used for evaluating static beam deflections. The presence of flexible restraints and elastic hinges is considered within the same framework. The application of the method is shown by some specific calculations.

From author's summary by H. C. Reggini, Argentina

1313. Chikov, E. A., Transverse vibrations of beams under the action of variable loads (in Russian), Report of the 16th Scientific Conference of the Professional Lecturing Staff of the Leningrad Engng.-Constr. Institute, Leningrad, 1958, 641–619; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5496.

A study is carried out of the joint action of longitudinal and transverse periodic forces on the vibrations and dynamic strength of a beam of constant section. For a beam with articulated supports the solution takes the form of

$$y(x, t) = T(t) \sin \frac{\pi x}{l}$$

The differential equation describing the motion of the beam with time is presented in the form of

$$\frac{d^2T}{dz^2} + (a - 2q \cos 2z)T = D\left(P_n + P_k \cos \frac{\eta}{\nu} z\right)$$

The solution is sought by means of the resolution of the unknown function T in a series by steps of the small parameter. The relations which were found show that the movement of each section of the beam takes place in conformity with the harmonic principle with periodically varying amplitude (pulsation). Pulsation also manifests itself in beams with different fastenings for the supports (a beam with one fastened and one free end and a beam with both ends fastened). For the cited cases the form of the vibrations approximate the function which satisfies the boundary conditions of the problem. The relation is ascertained between the frequencies

of the longitudinal and transverse forces at which loss of dynamic strength sets in.

V. A. Ivovich

Courtesy Referativnyi Zhurnal, USSR

1314. Kabulov, V. K., Investigations on the vibrations of a number of the simplest beam systems, using Volterra's integral equations (in Russian), *Dokladi Akad. Nauk UzSSR* no. 2, 5–10, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5499.

In order to ascertain the forces acting at various points in a vibrating beam system it is essential to know the transpositions of those points. This problem is solved by the author by bringing the differential equations for the longitudinal vibrations of the beams to the form of integral equations. The solutions for two of the simplest problems examined are not taken to the stage of numerical results, however. The inertia forces, arising from the transposition components in the direction perpendicular to the axis of the beam, are disregarded. Meanwhile these components possess, at ordinary values for the angles α and β, the same order of magnitude as in the longitudinal transpositions, while when the values for α and β are small they may even exceed them by many times. The transpositions of the inertia forces corresponding to the above, but not taken into account, are transferred in the form of support reactions to the nodes and alter the nodal loads and, consequently, the axial forces in the beams themselves.

A. V. Dyatlov

Courtesy Referativnyi Zhurnal, USSR

1315. Klees, G. T., Beam vibrations: Selecting beam dimensions to avoid undesirable resonance; determining fundamental frequencies of given beam configurations, *Mach. Design* **32,** 23, 205–207, Nov. 1960.

1316. Fedenko, G. I., Calculations for the vibrations of stepped girders of curvilinear beams (in Russian), *Sudostroenie* no. 7, 10–13, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5485.

A method is described for the determination of the frequencies of the free deflection vibrations of plane stepped girders and curvilinear beams. For a girder consisting of a number of rectilinear portions possessing variable stiffness and linear unit mass, it was established, on the basis of investigation by the method of deformation of a system of homogeneous equations, that the sum total of the characteristic numbers for all the portions with a determined form of fastening for the ends of the girder is the magnitude of the constant. For the purpose of ascertaining the frequencies of the deflection vibrations of the girder a formula having a simple form is furnished. A few numerical examples are examined of the determination of the frequencies of the free vibrations of plane beams having curvilinear and rectilinear portions.

K. V. Zakharov

Courtesy Referativnyi Zhurnal, USSR

1317. Brown, R. N., A torsional vibration problem as associated with synchronous motor driven machines, *ASME Trans.* **82** A (*J. Engng. Power*), 3, 215–220, July 1960.

Since synchronous motors have pulsating torque during starting cycle, torsional resonance may induce high stresses and fatigue failures. Author shows evaluation of resonant frequency, starting characteristics of motors, and evaluation of time at resonant torque during starting cycle. Methods of changing resonant frequency and starting characteristics of motor are discussed. Paper should assist in design of synchronous-driven high-inertia rotating equipment and is a warning not to overlook transient dynamic analysis in design.

Author overlooks effect of stress concentrations in drive mechanism. Failure under steady-state condition can be induced by rotating beam variation in bending stress with large stress concentrations.

K. H. Lenzen, USA

1318. Solecki, R., Free and forced vibration of a triangular plate (in Polish), *Rozprawy Inz.* **8,** 1, 65–81, 1960.

The triangular region considered is that of halved regular triangle. The deflection is expressed in the form of an infinite Fourier series. Free support on the entire edge and at one or several points is considered.

In the case of forced vibration a time-variable continuous load arbitrarily distributed over the plate is considered as well as a point load in the form of concentrated forces and moments.

The problem of nonhomogeneous boundary conditions is reduced to that of solving a Fredholm integral equation of the first kind.

A number of numerical examples and formulas for vibration modes and frequencies are given.

J. Kacprzynski, Poland

1319. Mazurkiewicz, Z., The boundary conditions and the equation of equilibrium and vibration for an anisotropic non-homogeneous plate (in English), *Arch. Mech. Stos.* **11,** 6, 729–735, 1959.

By means of Hamilton's principle and generalizing the results of his former paper ["Differential equation of equilibrium and vibration and expressions for the boundary conditions of an orthotropic non-homogeneous plate," *Arch. Mech. Stos.* **10,** 5, 755–757, 1958], author derives the equation of motion (equilibrium, in particular) and the boundary conditions for a thin nonhomogeneous anisotropic plate resting on a Winklerian foundation with variable foundation coefficient. The equations are derived in Cartesian coordinates. The equations for an orthotropic plate are derived as a particular case. The thickness varies symmetrically in relation to the middle plane. The load is assumed to be normal.

Z. Olesiak, Poland

1320. Borodachev, N. M., Vibrations of an elastic semi-space (in Russian), *Trudi Kubyshevsk. Inzh.-Stroit. In-ta* no. 5, 291–300, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5475.

The dynamic problem of a semispace ($y \geq 0$) is examined with the aid of Laplace's conversion on t and of a complex Fourier conversion on x. The equations for the motion are satisfied identically if the displacement components are expressed by known means through a Boussinesq bi-wave function. As the result of integral conversions the bi-wave equation changes into an ordinary differential equation for depicting the Boussinesq function, which is easily integrated. The formulas for the conversion in reverse enable the original to be recorded in integral form, which is then converted with the assistance of the correlations from Bessel's functions theory. The remaining two undetermined functions on which the original depends should be determined on the basis of the boundary conditions, as is shown in the example of the problems having these boundary conditions

$$u(x, 0, t) = 0, \nu(x, 0, t) = \varphi(t)\, e^{-a|x|}$$

I. S. Arzhanikh

Courtesy Referativnyi Zhurnal, USSR

1321. Bogomolov, S. I., Influence of the transverse vibrations of a turbine disc on the vibrations of the blades (in Russian), *Trudi Khar'kovsk. Politekhn. In-ta* **14,** 23–41, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5484.

An investigation is carried out of the joint transverse vibrations of a disk of constant thickness with blades of constant transverse section effecting independent deflection and torsion vibrations (the center of gravity of the transverse section of the blades coincides with the torsion center). The author produces an approximate solution for one example of the equation for the frequencies of the free vibrations of the disk with the blades. Results are given of the experimental determination on a model of the frequencies and

form of the vibrations of the disk with the blades, which satisfactorily confirm the calculation data. V. O. Bauer

Courtesy Referativnyi Zhurnal, USSR

1322. Isakson, G., and Eisley, J. G., Natural frequencies in bending of twisted rotating and nonrotating blades, NASA TN D-371, 67 pp., Mar. 1960.

An analysis and numerical results in the form of an extensive number of charts are presented to show the effect of twist on the natural frequencies of uniform and tapered nonrotating blades and the effect of twist and blade angle of the natural frequencies of rotating uniform blades. Offset of the root support from the axis of rotation in cantilever and articulated blades are considered.

H. H. Hilton, USA

1323. Kukuladze, M. N., Determination of the characteristics of the vibrations of rigidly fastened folded constructions by the use of V. Vlasov's method (in Russian), *Trudi Gruz. Politekhn. In-ta* no. 9 (57), 71–80, 1957; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5478.

The frequencies are determined of the free vibrations of thin-walled folded constructions, rigidly fastened along the transverse edges, with consideration and without consideration for the deformation of the contour of the transverse section. A method is utilized consisting of an eight-term set of differential equations and resolution by fundamental functions [V. Z. Vlasov, "The construction mechanics of thin-walled spatial systems," Moscow, Stroizdat, 1959]. A. K. Mroshchinskii

Courtesy Referativnyi Zhurnal, USSR

1324. Blekhman, I. I., and Dzhanelidze, G. Yu., Study of forced oscillations of some vibratory machines with many vibrators (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 3, 51–64, Mar. 1958.

This paper is concerned with the problem of forced oscillation of the basic elements of certain vibratory machines, such as vibratory conveyors, mechanical screens, vibratory platforms, and certain types of feeders having several individual vibrators distributed along the machine. Previous published works considered the working body of a vibratory machine as an absolute rigid body, whereas in this paper the vibrating member is considered as an elastic beam with evenly distributed mass. For normal operation, the vibrating member should oscillate to a certain approximation as a rigid body. However, this requirement is difficult to satisfy, since the natural frequency of transverse oscillations of these machines is often lower than the frequency of the operational oscillation. The solution to this requirement consists in the application of a relatively large number of low-power vibrators distributed along the machine with a small distance (pitch) between them. However, an excessive increase in the number of vibrators is undesirable for economical and technological reasons.

The authors suggest an analytical method for determining the magnitude of the flexural oscillations of the vibrating member relative to the member's basic (operational) transverse vibrations.

A formula was also developed for determining the maximum distance (pitch) between the sources of the disturbing force. This distance corresponds to the maximum permissible ratio between the amplitude of flexural transverse oscillation of vibrating member and amplitude of transverse components of the member's operational oscillations. E. I. Radzimovsky, USA

1325. Gofman, V. I., An investigation of the damping of vibrations by I. M. Rabinovich's geometrical method (in Russian), Investigations in the theory of constructions, no. 7, Moscow, Gosstroiizdat, 1957, 135–158; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5505.

The geometrical method evolved by I. M. Rabinovich for the solution of problems in the dynamics of ideally-elastic systems is generalized to include systems with damping proportional to the velocity of the motion. The vibration process is described by means of a curve (track-velocity). The solution is only given for a system with one step of freedom for the case of free and forced vibrations with a constant and harmonic exciting force.

M. L. Kempner

Courtesy Referativnyi Zhurnal

1326. Morrow, C. T., Application of mechanical impedance concept to shock and vibration testing, *Noise Control* **6,** 4, 5–9, July/Aug. 1960.

Book—Plunkett, R., (edited by), Colloquium on Mechanical impedance methods for mechanical vibrations, New York, American Society of Mechanical Engineers, 1958, 145 pp. $5.50. (Revs. 1327–1338)

Document is a compendium of papers presented at a colloquium sponsored by the Shock and Vibration Committee of the Applied Mechanics Division of ASME on 2 December 1958. Some papers are tutorial in nature, introducing mechanical impedance concepts, mechanical network theory, the statistics for treating random processes, and illustrating their applications in problems involving periodic steady state, transient, and random vibration. Others are concerned with methods of impedance measurement or computation, prediction of the statistical properties of large numbers of nominally identical single-degree-of-freedom systems, the presentation of impedance data on various structures, or the application of impedance concepts to the solution of specific vibration problems, e.g., the prediction of the effectiveness of foundation structures in reducing vibration, determination of the effects of nonrigidities in a vibration isolated equipment on the transmissibility of the equipment-mount system, determination of the critical vibration frequencies of steam turbines. A. O. Sykes, USA

1327. Plunkett, R., Introduction to mechanical impedance methods for vibration problems, 1–4.

Paper sets the stage for the balance of the colloquium by reviewing, historically, applications of mechanical impedance concepts to vibration problems, by discussing the definitions of impedance and mobility, the types of systems to which impedance concepts can be applied, and methods for their application. It also presents a relatively extensive bibliography which includes all mechanical impedance references cited in later papers in the colloquium. A. O. Sykes, USA

1328. Crandall, S. H., Impedance and mobility analysis of lumped parameter systems, 5–18.

Starting with Newton's law, Hooke's law and the definition of viscous damping, this tutorial paper develops the formulas for, and discusses the frequency dependence of the mobilities and impedances of lumped masses, springs, and dashpots. Following brief descriptions of force and motion generators, it discusses mechanical circuits, first defining point and transfer mobilities and impedances, and then studying a number of simple mechanical systems. After a section presenting circuit theorems, the paper concludes with an example illustrating the application of impedance methods to the determination of the effect on the natural frequencies of a crankshaft of adding a flywheel. A. O. Sykes, USA

1329. Wright, D. V., Impedance analysis of distributed mechanical systems, 19–42.

In vibration reduction problems in which it is desired to reduce machine-caused vibration in a floor structure, it is common practice to interpose, between the offending machine and the floor structure, a system of isolators (mounts) and a foundation.

This paper investigates the effects on the vibration of a floor structure of introducing simple foundations between it and the isolator system attached to the machine.

It discusses isolators and methods of representing them, suggests a mechanical circuit for certain simple floor structures, and tabulates formulas for the drive point impedances, transfer impedances, and force and velocity transfer ratios for columns, and center-driven pinned beams.

It compares the effectiveness of column, beam, and beam-column foundations in reducing floor vibration, and establishes a number of criteria useful in foundation design.

A. O. Sykes, USA

1330. Molloy, C. T., Four pole parameters in vibration analysis, 43–68.

The application to vibration problems of the mechanical equivalent of four-terminal electrical network theory is discussed. It is pointed out that the behavior of any vibrating linear elastic system with one input and one output terminal can be characterized by the equations:

$$F_{input} = \alpha_{11} F_{output} + \alpha_{12} V_{output}$$

$$V_{input} = \alpha_{21} F_{output} + \alpha_{22} V_{output}$$

where the F's are vibratory force amplitudes, the V's vibratory velocity amplitudes, and the α's, referred to as four pole parameters, are coefficients which define the mechanical characteristics of the system, the four pole, independent of whatever is attached to its terminals. A general procedure for obtaining the α's for any system is given, and then applied to their determination for rigid masses, massless springs, and massless viscous dampers. Rules for computing the α's for tandem, parallel, and tandem-parallel combinations of four poles are given and are illustrated by examples. The representation of mechanical four poles as electrical four-terminal networks is discussed, and both impedance and mobility analogs for the rigid mass, massless spring, massless damper, and the general four pole are given. After a section on the measurement of four pole parameters, the characterization of mechanical vibration sources is discussed. Four pole techniques are then applied to the vibration and shock isolation problems; their application to filters, shake tables and vibration pickups is described. Following an appendix which tabulates the four pole parameters for a variety of mechanical systems, the paper concludes with a bibliography listing pertinent references in the electrical, mechanical, and acoustical fields.

A. O. Sykes, USA

1331. Coleman, G. M., Methods for the measurement of mechanical impedance, 69–75.

The results of a survey on methods of measuring mechanical impedance are presented. Five methods for impedance measurement are described; the problems in which they have been applied and advantages and disadvantages of each are given. The methods included are:

1. The direct method in which a known vibratory force is applied and the amplitude and phase of the resulting vibratory velocity are measured.

2. A mechanical bridge method employing two simultaneously driven identical vibrating reeds, the free end of one of which is loaded with the unknown impedance, the value of which can be determined from measurements of the amplitude and phase of the vibratory displacements of the unknown and of the free end of the unloaded reed.

3. The motional impedance method in which the mechanical impedance of the unknown is deduced from electrical impedance measurements on a transducer attached to the unknown, and the properties of the transducer.

4. The substitution or reciprocity method in which the unknown mechanical impedance is deduced from the transfer responses of a mechanical driver and receiver with the unknown, and with a standard known impedance interposed between them.

5. The mechanical transmission line method in which the impedance terminating a transmission line is deduced from the ratio of the maximum velocity amplitude along the line to the minimum velocity amplitude, and the distance between the terminating impedance and the nearest minimum. A. O. Sykes, USA

1332. Neubert, V. H., and Ezell, W. H., Dynamic behavior of a foundation-like structure, 77–86.

The impedances of lumped mass-spring systems of only a few elements, and of simple continuous systems with known boundary conditions, e.g., rods, beams, plates, can be readily calculated. The impedance of built-up three-dimensional foundations composed, perhaps, of all of these elements, is much more difficult to obtain since measurements are usually required. An investigation of a simple three-dimensional foundation in which the continuous structure is replaced for calculation purposes by an equivalent system of masses and springs is presented. The theory is described; influence coefficients, resonant frequencies, and the drive point impedances of the structure in two directions at one corner, are calculated, measured experimentally, and compared. It is demonstrated that analytical prediction by high-speed computer of the impedance of simple three-dimensional structures is possible.

A. O. Sykes, USA

1333. Muster, D., Mechanical impedance measurements, 87–99.

Mechanical impedance data at a number of locations on the hulls of several U. S. submarines, at the midpoint of a beam mounted between two frames on a submarine hull, on a diesel engine subbase, on the air frame of a B47 plane, and a motor bed plate, and on a free-free beam are presented. The apparatus used in obtaining the data is described, and its limitations are pointed out.

The data should be useful to the general reader in familiarizing him with the sort of behavior one can expect from impedances of complicated structures. A. O. Sykes, USA

1334. Blake, R. E., and Belsheim, R. O., The significance of impedance in shock and vibration, 101–107.

The effects on design criteria for shock protection of not considering foundation and equipment impedance, when specifying shock motions, are discussed, and it is pointed out that overly conservative design criteria may result. The problems of computing the shock and random vibration responses of a system, given its sinusoidal response, and of estimating the effects on the response of a structure, to a given excitation, of adding a second structure are treated analytically using impedance concepts. The desirability of being able to control the output impedance of shock and vibration machines to better simulate actual environments is noted and an apparatus for achieving this result is suggested. Mechanical impedance measurement is discussed; a method for processing electrical vibratory force and velocity data, to obtain the real and imaginary parts of the mechanical impedance, is presented; and the impulse method for impedance measurement is described. A. O. Sykes, USA

1335. Ruzicka, J. R., and Cavanaugh, R. D., Vibration isolation of non-rigid bodies, 109–124.

The problem of isolating a nonrigid body is studied by investigating the transmissibility of an equipment-mount system simulated by a free-free beam driven at its center by a linear massless spring.

An analytical expression for the transmissibility of the system is derived, and calculations of the transmissibilities of three sys-

tems having beams with fundamental resonant frequencies 2, 5, and 10 times the natural frequency of the equivalent lumped mass-spring system are presented. These results are verified by comparison with experimental data. The conclusions are drawn that it is desirable to have the ratio of the fundamental resonant frequency of the beam to the natural frequency of the beam-mount system as high as possible and that this result can be achieved by making the mount stiffness as low as possible, the beam stiffness as high as possible.

In an appendix, four pole theory is reviewed, the relationships between the α four pole parameters and the transmissibilities and free and blocked impedances of arbitrary two-ended mechanical elements are established, and an expression for the mobility of a free-free beam is derived. A. O. Sykes, USA

1336. Chenea, P. F., and Bogdanoff, J. L., Impedance of some disordered systems, 125–128.

Spring constant, damping constant, and mass are considered as random variables in an investigation of the statistical properties of linear damped one-degree-of-freedom systems. The mean and variance of the impedances for both the mass-driven and spring-dashpot-driven systems and of the response function of the mass-driven system are calculated assuming that the random variable associated with the mass is rectangularly distributed and that the variance of each of the other two random variables is zero. The mean of the frequency response function for several values of the damping ratio is compared graphically with the frequency response of a deterministic system whose elements have values equal to the means of the random variables.

Expressions for the mean and variance of the impedance for the mass-driven system and calculations, in graphical form, of the mean and the variance of the impedance for the spring-dashpot-driven system for the case that spring constant, damping constant, and mass are random variables, for several values of damping ratio, for the three random variables independent and with identical distributions—a normal distribution in the latter case—are also presented. Results show that deviation of the means of the various quantities from their values based on a deterministic system, and their variances, are greater near the natural frequency of the deterministic system and are greater the smaller the damping; also that details of the random variable distributions influence the statistical properties of the systems quite strongly.

A. O. Sykes, USA

1337. Strasberg, M., Impedance concepts applied to mechanical systems excited by random vibration, 129–136.

Impedance concepts have long been applied to vibration problems in which the excitation forces and the responses to them are periodic. This paper discusses the application of the impedance concept to problems in random vibration. Necessary statistical concepts are defined, then utilized in the description of system response. The single-degree-of-freedom system is studied as an example. The physical significance of each statistical parameter is discussed, and methods for its measurement are described.

A. O. Sykes, USA

1338. Caruso, W. J., Prediction of critical speeds of steam turbines by dynamic stiffness methods, 137–145.

Critical speeds of turbines can be calculated provided the dynamic stiffnesses (or impedances), as functions of frequency, of the turbine rotors and the rotor support systems are known. This paper describes an investigation aimed at developing engineering information for calculating turbine criticals.

The first three criticals for a number of turbines were determined experimentally. The rotors were characterized dynamic-stiffness-wise by means of a calculation technique devised by Prohl. From the critical frequencies and the rotor dynamic stiffness data, the dynamic stiffness characteristics of the support system are derived. The results of the various tests are combined in a single design sheet from which, for turbines not too different from those studied, the first three criticals can be determined, if the rotor dynamic stiffness is known, to within about ±10%.

A. O. Sykes, USA

End of Symposium

Wave Motion and Impact in Solids

(See also Revs. 1216, 1276, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1380, 1393, 1522, 1534)

1339. Gorbatova, V. P., The intensities of refracted and reflected longitudinal waves with angles of incidence less than the boundary angles (in Russian), *Prikl. Geofizika* no. 18, 30–60, 1958; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 2946.

An approximate simplified method is given for the determination of the intensity of pure longitudinal waves propagating in ideally elastic horizontally-layered media; the influence exerted by the different parameters of the medium is investigated. It is assumed that the layers are sufficiently "thick," that the densities inside the layer are constant and change stepwise on the boundaries of the layers, and that the layers which have large longitudinal velocities of propagation also have high density and a large ratio of the transverse velocity to the longitudinal. It is proposed to determine the intensity of the waves in accordance with a somewhat scanty supply of curves furnished in the article. A method is put forward for calculating the intensity of the leading and the reflected (the plain and the short) waves. The leading waves are examined at points remote from their origin; the plain reflected waves, at points remote from the origin of the head waves, formed on the same division boundaries as the plain reflected wave being investigated. The short reflected waves are examined at points remote from the origin of similar reflected-refracted waves. An evaluation is furnished of the precision of the proposed method of calculation, permitting the establishment of boundaries at which the method can be justifiably employed. E. B. Vaserman

Courtesy Referativnyi Zhurnal, USSR

1340. Weber, C., The half-plane with a boundary having periodic waves and periodic concentrated forces and moments (in German), *ZAMM* **40**, 1/3, 14–21, Jan./Mar. 1960.

A semi-infinite region, taken to be under conditions of plane strain, has an edge of periodic wave shape, i.e. the edge is corrugated. The edge is subjected to a uniform distribution of isolated direct or shear forces or isolated couples applied at the successive crests of the corrugations, and there are appropriate reacting stress distributions applied at infinity. The stress analysis given of certain problems of the above class is based upon standard techniques. H. G. Hopkins, England

1341. Teodorescu, P. P., On the spatial problem of elastodynamics (in Roumanian), *Studii Si Cercetari Mecan. Apl.* **11**, 3, 681–694, 1960.

A formulation in stresses is given for the spatial problem of elastodynamics, with the aid of three functions each of them satisfying the equation

$$\Box_1 \Box_2 F(x, y, z, t) = 0 \quad \text{where} \quad \Box_{1,2} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c_{1,2}^2} \cdot \frac{\partial^2}{\partial t^2}.$$

Based on a theorem given by T. Boggio, the preceding equation is separated into two simple equations of longitudinal transversal

waves. The formulation in displacements is also discussed.

Results obtained generalize those corresponding to the static case, or those of the plane elastodynamics.

M. Soare, Roumania

1342. Buchanan, J. S., and James, H. J., Measurement of high intensity stress pulses, *Brit. J. Appl. Phys.* **10**, 6, 290–295, June 1959.

Explosive charge at one face of steel plate projects pellets attached at opposite face. Momentum and therefore stress magnitude and shape of pulse versus time are determined from pellet velocities.

Maximum tensile stress in fracture, determined from scabbing, results in linear function of maximum compressive stress, and in these tests (mild steel) higher than the static tensile stress by a factor about 6. Good reproducibility and discussion confer new value and interest to this old technique.

C. Riparbelli, USA

1343. Ivakin, B. N., The micro- and macrostructure of elastic waves in single-dimensional continuous heterogeneous media (in Russian), *Trudi Geofiz. In-ta Akad. Nauk SSSR* no. 39 (166), 92 pp. + illus., 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5459.

By the description "microstructure of waves in heterogeneous media" is understood the wave characteristics (velocity of propagation, changes in amplitude) determined on bases which are infinitely small or comparable with the length of the wave in the medium; this agrees with modern correlational seismic methods of prospecting for minerals. By macrostructure of waves is understood the wave characteristics measured on bases which are large or comparatively larger than the length of the wave in the medium. Detailed investigations are given in the monograph of single-dimensional problems regarding the micro- and macrostructure of plane waves, propagating in continuous three-chambered heterogeneous and generally absorbing media with one or two plane-parallel boundaries of division or with periodically recurring plane-parallel layers. It is assumed that the plane wave propagates in the direction normal to the layers of the medium. The possibility is indicated of solving wave problems for media with continuous variations of the elastic characteristics by means of the displacement method. A computing apparatus is adopted, on the basis of electro-mechanical analogy, for the theory of quadripoles and electrical lines. This theory enables the external (integral) wave characteristics of heterogeneous lines to be determined and to investigate the macrostructure of the waves. In order to determine the internal wave properties of the medium the author introduces a concept for a differential constant of propagation on an infinitely small base of a heterogeneous wave-conductor, on the assumption that the conductor's parameters are assigned. This concept enables the macrostructure of the propagating wave to be investigated. The solution of the wave problems examined is present in operational form. A detailed examination is made of a steady sinusoidal system of vibrations. N. A. Kil'chevskii

Courtesy Referativnyi Zhurnal, USSR

1344. Skuridin, G. A., An approximate solution of the problem on the diffraction of a plane elastic wave relative to a slot (in Russian), *Izv. Akad. Nauk SSSR, Ser. Geofiz* no. 1, 3–16, 1955; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5464.

Kirchhoff's method is generalized to include the diffraction of elastic waves and is supplemented by "the principle of the isolated element," according to which the plane wave is reflected from every point of the curvilinear boundary as well as from the plane element tangential to the boundary. The two-dimensional problem is investigated regarding the diffraction of a plane stationary longitudinal wave relative to a slot in an infinitely thin rigid shield, placed in an elastic medium. Diffraction produces longitudinal and transverse cylindrical waves, the amplitudes of which are distributed unevenly over the front. The dependence is computed of the amplitude on the angle of fall and on the position on the front in relation to the width of the slot and the length of the longitudinal wave, ~ 8. It is only possible to use the acoustic formulas for descriptions of the longitudinal waves in the region beyond the shield. However in the umbra zone the transverse and longitudinal waves possess an intensivity of the same order.

L. Malinovskaya

Courtesy Referativnyi Zhurnal, USSR

1345. Petrashen', G. I., Smirnova, N. S., and Gel'chinskii, B. Ya., Some problems in the dynamic theory of elasticity for media containing cylindrical or spherical boundaries of the division (in Russian), *Ucb. Zap. LGU, Ser Matem.* **27**, 170, 221–265, 1953; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5465.

Boundary problems in the theory of elasticity with assigned systems for the regions with cylindrical and spherical boundaries are solved by means of the method of incomplete division of the variables. It is explained that in the case of continuously distributed excitation the solutions are only classical concepts; should the reacting forces be concentrated the solutions will possess special features and can be used only as solutions in some general sense. The physical sense of the generalized solutions is discussed in detail. V. D. Kupradze

Courtesy Referativnyi Zhurnal, USSR

1346. Gurvich, A. I., Transverse impact of elastic bodies (in Russian), *Trudi Leningrad In-ta Inzh. Vodn. Transp.* no. 25, 134–140, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5491.

A solution is given for S. P. Timoshenko's known equation for the determination of an elastic contact force during impact of a body with a beam. The solution is sought in the form of a series resolved in steps of the small parameter. It is shown that the second approximation provides sufficient limitation. An analysis is made of the equation for the transverse vibrations of a doubly supported beam, subjected to impact by a sphere falling onto the center of the span. A numerical example is investigated of the determination of the contact force developed by a steel ball 2 cm in diameter when striking a steel beam of square section 1×1 cm and a length $l = 15.35$ cm with an initial impact velocity $v_0 = 1$ cm/sec. The results of these computations agree with the data of the solution of the same problem obtained by means of S. P. Timoshenko's numerical method. P. A. Stepin

Courtesy Referativnyi Zhurnal, USSR

1347. Tobe, T., Transverse impact on beams, *Bull. JSME* **3**, 10, 287–291, May 1960.

The problem of transverse noncentral impact of a solid circular cylinder on a beam is treated. A general integral equation is derived for impact at any point of a beam initially at rest. Various boundary conditions result in different sets of eigenfunctions to be considered. Assumed are the classical theory of beams and the Hertz law of impact. Difference between present paper and several previous ones on impact of quasi-spherical bodies lies in the different law of relative approach of the two bodies for a given force between them.

The integral equation obtained is a nonlinear Volterra equation with a Poisson-type kernel. It is solved in two ways, namely, by a step-by-step integration and by a collocation method, i.e., an approximation procedure which satisfies the integral equation at characteristic points. Primary and secondary impact curves are given in a numerical example, and the influence of four pertinent dimensionless parameters is presented in graph form.

D. C. Gazis, USA

1348. Sidlyar, M. M., A dynamic problem on the determination of stresses near an opening (in Russian), *Naucbn. Zap. Kievsk. In-ta* **16**, 16, 103–116, 1957; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5470.

An approximate investigation is made of the distribution of the dynamic stresses near an opening with free edges, situated in a rectangular plate being subjected to the action of longitudinal forces which show variations with time. It is assumed that the radius of the opening is small in comparison with the magnitude of the sides of the rectangular contour of the plate. The problem being investigated appears to be a generalization of the known statical problem ascribed to Kirsch. A solution is obtained by the method of operational computation by the application of Laplace's conversion and the investigation, together with the above, of the conversion of contour integrals. It is demonstrated that in the special case where the forces are suddenly applied to the contour, and eventually become constant, the maximum concentr ation of dynamic stresses exceeds the concentration of static stresses by a factor of 2.

N. A. Kil'chevskii

Courtesy Referativnyi Zhurnal, USSR

1349. Kartoshkin, L. I., Action of a momentarily applied inertia load on a rigid composite beam, having a stepped cross section (in Russian), *Izv. Akad. Nauk UzSSR Ser. Tekbn. Nauk* no. 4, 65–73, 1957; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5492.

This is an investigation of the action produced by a briefly applied finite impulse on a cantilever beam with a stepped cross section when being subjected to vibrations due to shear. Rigidity against deflection is taken to be infinitely large. Formulas are given, determining the beam's displacements, the inertia load and the tangential stresses. To illustrate a concrete example graphs are drawn of the accelerations, tangential stresses and the deflection curves for maximum acceleration and tangential stresses. For purposes of comparison the same type of graphs are drawn for a beam of constant section.

I. E. Dyshler

Courtesy Referativnyi Zhurnal, USSR

1350. Bagreev, V. V., Calculations for beams subjected to deflection impact (in Russian), *Trudi Mosk. In-ta Zh.-d. Transp.* no. 94, 143–149, 1957; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5494.

The contact force is determined, the force being produced by the impact of a sphere on the beam and account being taken of the local elasto-plastic deformations. To carry this out an approximate method is used, based on the solution of the known nonlinear functional equation of impact derived by S. P. Timoshenko by means of resolution of the nonlinear term into a series in steps of the small parameter. The resulting linear equation is solved by the use of the Bubnov-Galerkin method. A numerical example is examined which had been solved by Mason and Rochester by another method [*J. Appl. Mech.* **3**, no. 2, 1936]. The results of the two methods practically coincide.

P. A. Stepin

Courtesy Referativnyi Zhurnal, USSR

1351. Nowacki, W., Stress propagation in an infinite viscoelastic body produced by a time-variable point force (in English), *Arch. Mech. Stōs.* **11**, 6, 737–750, 1959.

An isotropic anelastic body is considered, with the relations: $\sigma = \sigma(\varepsilon)$:

$$P_1(D)\,P_3(D)\,\sigma_{ij}^{(1)} = P_2(D)\,P_3(D)\,\varepsilon_{ij}^{(1)} + \delta_{ij}\,\frac{1}{3}\,[P_1(D)\,P_4(D) - P_2(D)\,P_3(D)]\,\varepsilon_{kk}^{(1)} \quad [1]$$

(where P and D are operational polynomials in $\partial/\partial t$)

$$\sigma_{ij}^{(1)} = 2\int_0^t a(t-\tau)\,\frac{\partial}{\partial \tau}\,\varepsilon_{ij}^{(1)}\,d\tau + \delta_{ij}\int_0^t b(t-\tau)\,\frac{\partial\,\varepsilon_{kk}^{(1)}}{\partial \tau}\,d\tau\,. \quad [2]$$

The action of a concentrated force, a force uniformly distributed along a line or over a surface and a linear compression center is considered. Using the Laplace transformation for the time-variable and the Fourier integral for the geometrical variables, the system of equations of motion is solved, the solution for the viscoelastic body of the Biot type being obtained in a closed form.

S. Kaliski, Poland

1352. Blanchard, U. J., Landing-impact characteristics of load-alleviating struts on a model of a winged space vehicle, NASA TN D-541, 19 pp., Oct. 1960.

A brief experimental investigation was made of the landing-impact characteristics of a 1/9-scale dynamic model of a winged space vehicle. The landing tests were made by catapulting a free model onto a hard-surface runway and onto water. The model had a conical fuselage and a flat-plate wing with a basic delta planform and 75° sweepback of the leading edge. The use of yielding-metal shock absorbers and various landing-gear arrangements was investigated during landing impact.

The basic landing gear consisted of a dual rubber-tired nose wheel and twin main skids aft of the center of gravity near the wing tips. Landing motion and acceleration data were obtained over a range of landing attitudes, gross weights, and initial sinking speeds. Brief tests were made with an alternate nose-wheel location. An all-skid configuration also was briefly evaluated for hard-surface and water landings.

The landing gear employing yielding struts for impact-energy absorption during hard-surface landings resulted in accelerations of approximately 5½g near the nose gear over a range of landing parameters. Replacing the nose wheel and tire with a skid did not significantly change the accelerations. Landings in smooth water with rigid struts and adequate planing area at the nose skid resulted in a maximum landing acceleration of approximately 4 g.

From author's summary

1353. Nowacki, W., and Sokolowski, M., Propagation of thermoelastic waves in plates (in English), *Arch. Mech. Stos.* **11**, 6, 715–727, 1959.

Authors consider the problem of propagation of plane thermoelastic waves in isotropic plates of various thicknesses. The problem under consideration is described by a system of differential equations in the form given by M. A. Biot. The boundary conditions of constant temperature and perfect heat insulation are assumed for surfaces free from stresses.

Some simplifying assumptions are made, thus enabling the problem to be reduced to a form such that the computation of examples is relatively easy. The influence of coupling on the character of propagation of elastic waves is discussed in two extreme cases: a very thick and a very thin plate in relation to the wavelength. This influence is twofold: it increases the phase velocity of the wave motion and it makes dispersion terms appear in the solutions for displacements.

Z. Olesiak, Poland

1354. Miklowitz, J., Plane-stress unloading waves emanating from a suddenly punched hole in a stretched elastic plate, *ASME Trans.* **82** E (*J. Appl. Mech.*). 1, 165–171, Mar. 1960.

Using the principle of Kromm's solution for the plane stress problem for compressional waves in an infinite elastic plate, subjected to a sudden radial pressure or radial velocity in a circular hole at its center, the author derives solution of the mentioned problem. The Laplace transformation of the governing partial differential equation with respect to initial conditions leads to normal Bessel differential equation. Further solution is provided by means of Mellin's inversion integral and contour integration. Numerical results were obtained employing an electronic computer and are in good agreement with those in Kromm's and Selberg's work. Paper is of interest in fragmentation studies and forms the first approximation of the problem of propagation of cracks caused by punching in large pressurized tanks.

V. Kopriva, Czechoslovakia

Soil Mechanics: Fundamental

(See also Revs. 1364, 1393)

Book—1355. Sokolovski, V. V., Statics of soil media (Translated from the Russian 2nd ed. by D. H. Jones and A. N. Schofield), Toronto, Canada, Butterworth & Co., Ltd. 1960, xiii + 237 pp. $10.50.

Writer gives the known theory of critical equilibrium of soil media, which is explained in detail, both for granular and cohesive soils. The theory is applied to the stability of foundations and slopes, and to the pressure against retaining walls. For different cases tables and graphs are given to facilitate calculation. Writer oversimplifies, however, the behavior of soil media, assuming zero deformation during loading until critical equilibrium (c.e.). Thus one might be tempted to admit a certain part of the calculated c.e. on the soil, according to the use in other construction fields. This may be approximately all right with rather stiff soils; in soft soils deformations may become inadmissible long before a so-called safe part of the c.e. is reached. Furthermore these deformations may, in some soils, even have influence on the c.e. itself, e.g. in the case of "quick" clays, where the deformations cause diminution of the shearing resistance and subsequently of the c.e. Care should therefore be taken, in applying the given method, to take the soil properties into account.

F. C. De Nie, Holland

1356. Haythornthwaite, R. M., Stress and strain in soils, Brown Univ., Div. Engng. TR-3 (Contract DA-19-020-ORD-4566), 18 pp., May 1960.

Some results obtained in the course of an experimental program are described. Tests of a remolded silt formed into hollow cylinders and subjected to combined axial compression and torsion in the presence of an all-round hydrostatic pressure indicate what appear to be significant departures from the usual generalization of the Coulomb yield condition for complex stress states. Moreover, with the exception of stress states close to a simple compression superimposed on a hydrostatic pressure, the directions of flow followed closely those normally associated with an ideally plastic body which obeys Tresca's yield condition. Both the yield stresses and the flow directions appeared to be independent of the loading path. From author's summary

1357. Voitkovskii, K. F., Creep of frozen soils when under the action of pure shear (in Russian), Trudi Ser.-Vost. Otd. In-ta Merzlotovod. Akad. Nauk SSSR no. 1, 35-45, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5624.

The equipment used for the investigation of the creep of frozen soils is described; the method employed was the twisting of soil samples in the form of cylinders. Samples of sandy and loamy soils were tested at tangential stresses of 0.25-2 kg/cm^2. It was established that there is a limit for long-term strength of frozen soils above which the deformation increases continuously. The creep curves obtained satisfy the equations derived by S. S. Vyalov [*Dokladi Akad. Nauk SSSR (N.S.)* **104,** 6, 850-853, 1955; **111,** 6, 1193-1196, 1956]. The limits for short-term and long-term strength for the loamy soil proved to be higher than for the frozen sandy soil. N. I. Malinin

Courtesy Referativnyi Zhurnal, USSR

1358. Gol'dstein, M. N., and Gol'dberg, P. Ya., Strength of loess soils (in Russian), *Gidrotekh. Stroit.* **27,** 4, 39-42, Apr. 1958.

Laboratory tests have been made with loess samples the results of which permit the following conclusions:

The wetting of the loess at any state of stress causes a sudden decrease of its shearing strength. The strength disappears completely at zero pressure and increases with increasing hydrostatic pressure as well as with decreasing dielectric constant of the wetting liquid. The strain increases with increasing difference between the principal stresses. The strength of wetted loess samples depends also upon the polarity of the wetting liquid, upon its chemical composition as well as upon the character of the salt and clay binder. L. Suklje, Yugoslavia

1359. Forecasting soil behavior under load, *Engineering* **190,** 4919, p. 156, July 1960.

Soil Mechanics: Applied

(See also Revs. 1249, 1355, 1357, 1358, 1393)

1360. Hansen, B., Limit design of pile foundations (in English), *Bygnsstat. Medd.* **30,** 2, 37-86, Sept. 1959.

An entirely general method of design based on theory of plasticity which extends the work of Nokkentved and Vandepitte, and including "shakedown" failure by wriggling down into the soil due to continuing cycles of plastic deformation, as well as local and total failures. Method is especially useful for three-dimensional pile groups and covers elastic, elastic-plastic, and ultimate states. Technique of computation is explained and systematized, suggesting application to electronic calculating machines. Existing methods are reviewed and critically evaluated, with special emphasis on the merits of the well-known Culmann method, and are related to the general method proposed. Reviewer considers paper an important contribution demanding the attention of any serious student of soil mechanics and foundations. Paper is lucid, well-written, and technically sound.

G. P. Fisher, USA

1361. Hansen, B., A new design method for pile foundations (in English), *Bygnsstat. Medd.* **31,** 1, 1-29, Feb. 1960.

Limit design of pile foundations is explained in two examples, plane and three-dimensional respectively. Method used was published earlier [see preceding review]. In this paper only summary of method is given. It is assumed that rigid foundation body is supported on batter piles with hinges on tops and points. Supporting foundation with use of five elastically loaded piles in three-dimensional case or with two piles in plane case we obtain system with one degree of freedom which enables other piles to be loaded to failure. This assumption gives the first criterion. Nevertheless, changes of loading diminish bearing capacity of piles to be loaded to failure. To include this, a second criterion "shakedown problem" is defined, requiring greater number of piles than first criterion.

Reviewer believes experiments, verifying validity of both criterions under different soil conditions and ascertaining if there are not great settlements, are desirable to justify the general use of the more economical limit-design method, in place of elastic methods [see Asplund, S. O., AMR **10**(1957), Rev. 2886].

Z. J. Bazant, Jr., Czechoslovakia

1362. Kroie, A., Practical design of pile groups, *Struct. Engr.* **38,** 7, 230-234, July 1960.

Pile loads are calculated assuming that all piles are end bearing with no yielding at the ends and no side friction or bending of piles. Pile cap is assumed rigid. The statement "treating (a statically indeterminate pile group) as a redundant frame will give contradictory results depending on which member is taken as redundant" is, in the opinion of the reviewer, incorrect.

M. L. Pei, USA

1363. Bolcskei, E., and Domjan, J., Foundations with small settlement (in Hungarian), *Mélyépítéstudományi Szemle* **10,** 4, 166-171, Apr. 1960.

1364. Chuang, L. H., Calculation of bearing capacity of footings (in Chinese), *Chinese J. Civ. Engng.* **6**, 5, 382–391, May 1959.

The bearing capacity of an infinitely long footing resting on soil with cohesion c and angle of internal friction ϕ is discussed. The derivation follows generally that of Prandtl and Terzaghi. Weight of soil within the failure zone is considered and logarithmic spiral slip surface is assumed. The resulting formula is of the form (using Terzaghi's notation) $q_u = cN_c + \gamma D_f N_q + \gamma b N_\gamma$.

Values of the three coefficients given in the paper are within 20% of the corresponding coefficients given by Terzaghi for the general shear case. Local shear case is not considered. All seven references listed are in Russian or Chinese.

M. L. Pei, USA

1365. Akutin, G. K., An investigation of the compacted zone after carrying out work in soft soils by compacting them by the use of the energy derived from blasting (in Russian), Questions arising in the opening-up and exploitation of deposits of minerals. Kiev, Akad.Nauk USSR, 1958, 31–36; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5607.

Experimental curves are given to show the relation of the volumetric weight and the resistance in terms of time to the crushing of the soil (clay, loam) within the limits of the zone, where compacting has taken place as the result of the detonation of a drill charge. Simple empirical formulas are also furnished for the determination of the thickness and of the external radius of the compacted zone. The compacted zone is divided into an inner and an outer one: empirical formulas and curves are provided to indicate the relations between the radii of these zones and the exploitation radius and the radius for the charge. G. S. Grigoryan

Courtesy Referativnyi Zhurnal, USSR

1366. Efremovich, L. A., and Sokolov, A. G., An investigation on the work of an anchor-plate and the procedure for its computation (in Russian), Information data for steel constructions, **1**, Moscow 1957, 88–103; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5612.

A brief description is given of an anchor-plate used to anchor in the soil the guy ropes of radio-masts, of bridges and of masts for special purposes. This system is an economical piece of equipment compared with the massive bulk of anchoring foundations used. Basic formulas are furnished for the anchor-plate, which have been derived from computations adopted for supporting walls; the consideration given to the friction forces on the lateral surfaces of the soil mass is explained and a simplified formula is provided. The calculations for a vertical round anchor are described. These are based on the assumption that the wedging-out of the soil if the anchorage failed would take place along the curve of the surface of rotation, which depends on the angle of internal friction of the soil and is determinable, moreover, by the minimum force required to tear out the anchor; the curve of the generatrix of the surface of wedging-out is assumed to be a parabola. The extension of this concept of the matter to include an inclined anchor and an arbitrary form for the plate in plane resulted in calculation formulas which took into account both friction and cohesion. For the purpose of effecting agreement between the formulas and the results of the experiments it was found necessary to introduce a simplification factor. The experiments are described of investigations on model plates with dimensions of 50 × 50, 50 × 75, 50 × 100 and 50 × 125 mm and a diameter of 50 mm, placed in soil contained in a not very large box, and buried at various depths in the soil. Tests were also carried out with plates measuring 400 × 400 and 400 × 800 mm embedded 80 cm deep in a loess type of soil. The data collected by E. Z. Yasin on the tearing-out of linear anchors in the form of round bars are published and a comparison is given of the actual resistance of the anchors with the calculated, using the proposed formulas.

G. A. Savitskii

Courtesy Referativnyi Zhurnal, USSR

1367. Dick, T. M., and Brebner, A., A laboratory study of pneumatic breakwaters, Queen's Univ., Ontario, Civ. Engng. Dept. Rep. no. 12, 31 pp., July 1960.

1368. Kogan, B. I., Computations for the strength of asphalt-concrete road coverings (in Russian), *Trudi Mosk. Avtomob.-dor In-ta* no. 23, 127–133, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5619.

The criterion for the strength of the construction is taken to be the magnitude of the maximum permissible vertical sag of the top dressing of the road. This was also the case in N. N. Ivanov's method and others ["Designing of road top dressings," Moscow, Avtotransizdat, 1955]. The sagging of a multilayer construction is determined as the sagging of a two-layer system with the modulus of deformation of the top dressing continuously changing with depth, described by means of an exponential function, two constant parameters which are chosen in accordance with the condition of the equality of the areas of the diagrams of the moduli of deformation of the multilayer system and of the layer with a modulus continuously changing with depth. The rigidity of the two-layer system is evaluated by means of the equivalent modulus of deformation of a homogeneous semi-space. As a supplement to the calculation by N. N. Ivanov's criterion the recommendation is made to carry out the calculation for the asphalt concrete layer by the maximum tension stresses. The proposed relations are compiled into nomograms. Two numerical examples are given.

I. I. Kandaurov

Courtesy Referativnyi Zhurnal, USSR

Processing of Metals and Other Materials

(See Revs. 1242, 1392, 1559)

Fracture (Including Fatigue)

(See also Revs. 1271, 1317, 1342, 1559)

1369. Barenblatt, G. I., Equilibrium cracks formed during brittle fracture. Rectilinear cracks in plane plates, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **23**, 4, 1009–1029, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

The problem of cracks, the dimensions of which do not change at a given load, in the case of plane plates is considered. Some basic concepts developed in a previous paper [*Prikl. Mat. Mekh.* **23**, no. 3, 1959] are also used. The dimensions of the crack are again determined by the applied loads and by a new universal characteristic coefficient of the material, called "cohesion modulus". It is shown that the cohesive forces are of essential influence only on the dimensions of the cracks and on the distribution of the displacements of the opposite edges of the crack in the vicinity of its ends. The problem of driving a rigid wedge in an infinite plate of brittle material is studied.

N. E. Cristescu, Roumania

1370. Starodubov, K. F., and Polyakov, S. N., The tempering brittleness of carbon steel (in Russian), *Izv. Vyssh. Uchebn. Zavedenii, Ferrous Metallurgy* no. 3, 131–144, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5805.

It is shown that carbon steel of the type used in wheels (railway) and in rails possesses susceptibility to tempering brittleness in the range of temperatures between 500 and 650°. The embrittlement of the steel develops to its maximum degree after a high tempering at a cooling of 20°/hr. Real influence on the inclination of

carbon steel to tempering brittleness is exercised by manganese and phosphorus. Especially injurious is the joint increase in content of these elements. As the result of investigations of the hardness, coercive forces and electro-resistance the fact is confirmed that the basis of the embrittlement of steel lies in the separation of particles of a new phase along the grain boundaries.

N. M. Dubrovin
Courtesy Referativnyi Zburnal, USSR

1371. Whaley, R. E., and Kurzhals, P. R., Fatigue-crack propagation in aluminum-alloy tension panels, NASA TN D-543, 25 pp., Nov. 1960.

Results are presented of a series of fatigue tests to study crack propagation and the resulting stress distributions in tension panels. The panels were all of the same general design, and configurations varied mainly in the relative amount of cross-sectional area in the skin, stiffeners, anf flanges. The panels were constructed of 2024-T3 and 7075-T6 aluminum alloys. It was found that the average rate of crack growth was slower in panels made of 2024-T3 aluminum alloy than in panels made of 7075-T6 aluminum alloy. All cracks initiated in the skin, and the slowest crack growth was measured in configurations where the highest percentage of cross-sectional area was in the stiffeners.

Strain-gage surveys were made to determine the redistribution of stress as the crack grew across the panels. As a crack approached a given point in the skin, the stress at that point increased rapidly. The stress in the stiffeners also increased as the crack approached the stiffeners. During the propagation of the crack the stress was not distributed uniformly in the remaining area.

From authors' summary

1372. Smith, R. W., and Smith, G. T., Thermal-fatigue crack-growth characteristics and mechanical strain cycling behavior of A-286, Discaloy, and 16-25-6 austenitic steels, NASA TN D-479, 67 pp., Oct. 1960.

Thermal-fatigue crack-growth characteristics of notched- and unnotched-disk specimens of A-286, Discaloy, hot-cold worked 16-25-6, and overaged 16-25-6 were experimentally studied. Separately controlled variables were total strain range (0.0043 to 0.0079 in./in.), maximum cycle temperature (1300° and 1100° F), and hold time at maximum temperature (0 and 5 min). A limited number of mechanical, push-pull, constant-strain cycle tests at room temperature were made using notched and unnotched bars of the same materials. In these tests the number of cycles to failure as well as the variation of load change with accumulated cycles was measured, and the effects of mean stress were observed.

Constant-strain-range mechanical-fatigue tests at room temperature revealed notched-bar fatigue life to be strongly influenced by mean stress. For a specific strain range, the longest fatigue life was always found to be associated with the least-tensile (or most compressive) mean stress.

By defining thermal-fatigue life as the number of cycles required to produce a crack area of 6000 square mils, the relative thermal-fatigue resistances of the test materials were established. Notched-disk specimens of A-286 and Discaloy steels exhibited longer fatiuge lives than either hot-cold worked or overaged 16-25-6. On the other hand, unnotched-disk specimens of Discaloy and hot-cold worked 16-25-6 had longer lives than A-286 and overaged 16-25-6. Separation of the crack-growth data into microstage and macrostage periods revealed that the macrostage period accounted for the greatest part of the difference among materials when tested in the notched configuration, while the microstage was largely responsible for the differences encountered in unnotched disks.

From authors' summary

1373. Shal'nev, K. K., Hydromechanical aspects of cavitational erosion (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 1, 52-62, Jan. 1958.

Author briefly reviews some literature on cavitation. Shal'nev's interest in cavitational phenomena lies mostly in the degree of cavitational erosion. The evaluation of cavitation and resulting cavitational erosion are presented in terms of the following parameters: cavitational index X (also known as cavitational parameters), length of cavitational erosion λ; Reynolds number N_{Re}; volume of erosion ΔV.

The cavitational erosion is investigated on lead plates 3 mm and 8 mm thick. The "set-up" of apparatus and the research procedure are outlined in some detail.

The results are presented in terms of graphs; for example, velocity of a stream ft/sec versus erosional volume ΔV, or erosional volume versus R_N; or in terms of specific volume ΔV_s versus R_N.

N. V. Feodoroff, USA

1374. Leybold, H. A., Axial-load fatigue tests on 17-7 PH stainless steel under constant-amplitude loading, NASA TN D-439, 17 pp., Oct. 1960.

Axial-load fatigue tests were conducted at room temperature on notched and unnotched sheet specimens of 17-7 PH stainless steel in Condition TH 1050. The notched specimens had theoretical stress-concentration factors of 2.32, 4.00, and 5.00. All specimens were tested under completely reversed loading. S-N curves are presented for each specimen configuration and ratios of fatigue strengths of unnotched specimens to those of notched specimens are given. Predictions of the fatigue behavior of notched specimens near the fatigue limit were made.

From author's summary

1375. Foster, L. R., Jr., and Whaley, R. E., Fatigue investigation of full-scale transport-airplane wings—Tests with constant-amplitude and variable-amplitude loading schedules, NASA TN D-547, 37 pp., Oct. 1960.

Six fatigue tests were conducted on C-46 airplane wings: Two tests at a high constant-amplitude load, two tests with loading schedules based on gust-frequency statistics, and two tests with loading schedules developed from maneuver-loads statistics. The results are presented, discussed, and compared with previously published data from other tests in this series. Although constant-amplitude tests at high load levels tended to produce cracks at the same locations as did tests at variable amplitude, the critical crack of variable-amplitude tests was not defined by tests at constant amplitude.

From authors' summary

1376. Taira, S., and Koterazawa, R., Influence of frequency of stress cycles on fatigue at elevated temperatures, *Bull. JSME* **3,** 10, 235-241, May 1960.

Constant deflection amplitude bending fatigue tests at 450°C are described for an annealed 0.55% carbon steel and a quenched and tempered 13% chromium steel, at each of two frequencies, 170 and 1500 cpm. Results are compared with those at selected higher temperatures and with previous work on a low-carbon steel. Below a critical temperature appropriate to the material, the low frequency tests endured more cycles to fracture than the high frequency tests. Above this temperature the reverse tendency occured, the lives at each frequency converging at high strain amplitude. Above the critical temperature the life in hours at each frequency tended to converge for small strain amplitudes.

Tentative explanation of this behavior is put forward in terms of metal physics concepts, by suggesting that, for a given temperature, time to fracture is independent of frequency for a constant true stress amplitude but cycles to fracture is independent of frequency for constant plastic straining action. Some support for the theory derived is gained from analysis of earlier results of other workers.

C. E. Turner, England

1377. Wilkins, E. W. C., Some problems of stress concentration in aeronautical construction elements subjected to fatigue (in French), *Rev. Métall.* **56**, 3, 316–326, Aug. 1959.

Study of the influence of stress concentration on endurance limit, stress concentration factor being determined by photoelasticity for several important cases in aeronautical construction. Author enphasizes the limited value of tensile strength alone to the designer of parts under repeated stresses. Suggestion is made as to the number of cycles in a fatigue test on which the conventional endurance limit should be based for steel and nonferrous alloys used in aeronautical construction.

J. A. Cintra, Brazil

1378. Korchinskii, I. L., The carrying capacity of materials when subjected to a small number of repeated loads (in Russian), *Byul. Stroit. Tekhn.* no. 3, 19–22, 1958; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 3329.

The graph indicating the connection of the logarithm of the number of cycles of loading and the carrying capacity of the material represents approximately a broken line consisting of two straight lines. The first straight line is inclined, having an initial ordinate equal to the carrying capacity for momentary loading (the static limit of strength). The second straight line is horizontal, with ordinates equal to the limit of fatigue. A point is recommended for use as the point of fracture on the graph, corresponding to the limit of fatigue with $n = 2 \times 10^6$ cycles. Working formulas are furnished for calculating the carrying capacity of brittle and plastic materials for 100-times and 1000-times repeated loadings.

A. D. Pospelov
Courtesy Referativnyi Zhurnal, USSR

1379. Gokhberg, M. M., The fatigue strength of the metallic structural parts in cranes (in Russian), Questions in the theory and calculations of hoisting and transporting machines, Moscow-Leningrad, Mashgiz, 1957, 105–122; *Ref. Zh. Mekh.* no. 5, 1959, Rev, 5721.

A procedure is proposed for the dynamic calculations of crane constructions based on the following assumptions: (1) the damping due to the action of forces of resistance is disregarded, (2) the number of revolutions of the electro-motor is taken to be constant, (3) the elasticity of the components of the transmission system is not taken into account and the construction of the crane is investigated as a system with one step of freedom. Two stages in the work of the crane are investigated: (a) the load had not yet been detached from the ground; (b) when the freely suspended load is moving. Recommendations are given in regard to the calculations of crane constructions for durability; values are furnished for the effective coefficients of stress concentration. Information is given regarding the selection of a coefficient for the factor of safety.

S. V. Boyarshinov
Courtesy Referativnyi Zhurnal, USSR

1380. Davidenkov, N. N., and Belyaeva, E. I., Investigation of impact fatigue (in Russian), *Metallovedenie i Obrabotka Metallov* no. 9, 12–15, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5821.

The limit of fatigue for steel when subjected to cyclic impact loading differs, according to the experimental data, from the static; it is noted further that it gets lower in proportion to the reduced stability state of the steel.

From authors' summary
Courtesy Referativnyi Zhurnal, USSR

1381. Shchepkin, G. V., A graphoanalytical method for calculating the endurance of shafts (in Russian), *Trudi Khar'kovsk. Politekhn.* In-ta **16**, 41–49, 1957; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 3252.

A method is proposed for calculating shafts, taking into account the stresses and the dimensions factor in accordance with known magnitudes of the deflectional and torsional moments. Normal stresses are assumed to change by the symmetrical, while the tangential vary by the pulsation, cycle. Tables are given for evaluating the effective coefficients of concentration (for the fillets and the cotter). Numerical examples are furnished.

I. I. Trapezin
Courtesy Referativnyi Zhurnal, USSR

Experimental Stress Analysis

(See also Revs. 1257, 1271, 1306, 1401)

Book—1382. Gohlke, W., Introduction to piezoelectric measuring methods, 2nd ed. [Einfuhrung in die Piezoelektrische Messtechnik], Leipzig, Akademische Verlagsgesellschaft Geest und Portig K.-G., 1959, 278 pp. + 219 fig. + 8 tables + 223 refs.

This is a comprehensive treatment of a subject which has an ever-growing importance for the further development of internal combustion engines, gas turbines, and other structures subjected to rapidly fluctuating forces. It covers completely, clearly, and understandably the subject indicated in its title. The basic concepts of piezoelectric phenomena, the history of piezoelectricity, and detailed characteristics, are described and explained. Application to pressure measurement is treated in detail; numerous pressure pickups are illustrated and described, and their static and dynamic properties discussed. Error sources, such as the influence of connecting passages for the pressure gases, and the mechanical vibrations of the pickup itself are discussed, and methods for their compensation or mitigation are suggested.

A section deals with design features of pickups for use on internal combustion engines, and on ballistic devices. Another section treats transmitters for measurement of force and acceleration. Elements of electric circuitry, such as amplifiers and their frequency characteristics, oscillographs of the mirror and of the cathode-ray type, are treated, with their auxiliary equipment, such as power sources. Calibration and testing of transmitters and of their circuit elements are explained. Auxiliary equipment, such as crank-angle transmitter, and piston-stroke transmitter, is illustrated and described. Special adaptations for measuring pressure differences, knocking phenomena (very rapid pressure rise), ignition delay, maximum pressure, are treated, with illustrations and descriptions; also some photoelectric devices are included. Disturbances of functioning are discussed. Characteristics of various piezoelectric crystals, Seignette salt crystals (Rochelle salt), and polycrystalline piezoelectric ceramics are given, with diagrams.

All this information is given in a systematic, well-prepared manner, giving theoretical background and examples of practical executions, and in many cases with experimental results. The extensive reference list has been collected mainly from German, but also from American, British, French, and Russian, sources. This is a well-rounded, informative and up-to-date book, which should be of considerable usefulness for designers and users of piezoelectric devices.

K. J. DeJuhasz, USA

1383. Hiltscher, R., and Muller, R. K., Analysis of the concrete reinforcement of steel construction with the aid of photoelasticity (in German), *Beton u. Stahlbeton.* **54**, 11, 263–271, Nov. 1959.

The paper deals with experimental techniques for constructing a composite model of a photoelastic plastic with embedded aluminum wires to simulate reinforced-concrete-steel structures. A bending beam made of this composite material was used for calibration purposes. Plates with different boundary conditions have been investigated. Simulation of a crack induced in the structure and the resulting stress pattern is discussed in some detail.

In all examples, assumption was made that the structure was subjected to plane stress. In addition to shear stress distribution as obtained through photoelastic analysis, separation of principal stresses is done by a lateral extensometer and mechanical strain gages.

It appears that the technique proves satisfactory for stress analysis of composite models for reinforced-concrete-structure representation. A summary of calculations for reinforced-concrete structures is presented and its validity discussed.

The paper is well written and the theme is well developed. It is intended for researchers in the fields of photoelasticity and reinforced concrete. F. Zandman, USA

1384. Zellerer, E., and Thiel, H., Force field in a wall-beam on two supports with three door openings in the case of unsymmetric concentrated forces (in German), *Bautechnik* **36**, 3, 84-95, Mar. 1959.

Standard two-dimensional photoelastic methods are applied to a wall panel containing three door openings. The panel is on two supports and is loaded unsymmetrically in its plane by two concentrated loads.

Two photoelastic models were constructed 1/50 actual size. The first, for isochromatic photographs, was of VP 1527 material. The second, for observation of isoclinics, was of less optically sensitive Plexiglas. Both models were 1 cm in thickness.

Ten locations—the end sections, sections between door openings, and sections above and below each door opening—are examined in detail for shear and normal stress distribution. Detailed stress diagrams are shown for each of the sections and a composite diagram for the entire panel gives resultant shear and normal forces at each section.

Frocht's shear-difference-method is used for calculation of stresses at interior points. Lateral extensometers (after Hiltscher, 1957) are said to give a close experimental check.

A. G. Sharp, USA

1385. Buhl, J. E., Jr., and Brock, J. S., A photoelastic investigation of stresses in the hub of a proposed 20,000-SHP controllable-pitch propeller, David W. Taylor Mod. Basin Rep. 1417, 17 pp., July 1960.

A three-dimensional photoelastic model of the hub of a proposed 20,000-shaft-horsepower controllable-pitch marine propeller was built and tested. Stresses were determined by strain gages and photoelastic measurements at selected locations for the "crash-stop" load condition.

The hub stresses for the case of one blade hitting a submerged object were investigated to a limited extent by strain gages.

Stress concentrations at the inside corners were extremely high, causing stresses of the order of 65,000 psi to 84,000 psi in the prototype. It is recommended that larger fillet radii be used at all inside corners to reduce the stresses.

From authors' summary

1386. Schumann, W., Fundamentals of three-dimensional photoelasticity (in German), *Schweiz. Bauztg.* **78**, 47, 755-761, Nov. 1960.

1387. Moody, E. S., and Denehy, D. R., An unbonded electrical resistance strain gauge force transducer, Aero. Res. Lab., Melbourne, Austral., Note SM 266, 7 pp. + 3 figs., Apr. 1960.

1388. Biderman, V. L., and Pugin, V. A., A wire strain gage for the measurement of large deformations (in Russian), *Zavod. Lab.* **24**, 7, 874-875, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5881.

The gage consists of a rubber filament of 1-1.5 mm diameter onto which is wound fairly closely a thin constantan wire.

From authors' summary
Courtesy Referativnyi Zhurnal, USSR

1389. Pustynnikov, V. G., Measurement of momentary deformations without the use of an oscillograph (in Russian), *Trudi Rostovsk.-n/D. In-ta S.-kh. Masbinostr.* no. 8, 363-376, 1957; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 3410.

A schematic electric plan, based on the carrier frequency, is proposed with whose aid it is possible to investigate deformations in the moving components at given fixed positions for the components. Instead of an oscillograph as the measuring instrument the proposal is made to use a robustly constructed milliameter capable of standing up to knocks and shaking up. A device of this nature, in the author's view, can be used to measure dynamic deformations varying with a frequency of 3 c/s and higher.

A. D. Pospelov
Courtesy Referativnyi Zhurnal, USSR

1390. Dean, D. S., An improved strain-gauge type of load-cell thrust transducer, Aero. Res. Counc. Lond. Rep. Mem. 3153, 12 pp., 1960.

1391. Melent'ev, P. V., A precise investigation of plastic deformations (in Russian), *Sb. Leningrad In-ta Inzh. Zh.-d. Transp.* no. 156, 139-166, 1958; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 3413.

Two pieces of apparatus designed by the author are proposed for this investigation: an extensometer and a torsiometer (acoustic). Plans and descriptions of the apparatus are given and the features special to their working are noted. Author states that the extensometer is capable of measuring deformations of the order of from 0.2 to 20 μ/sec (with a base of 100 mm). The torsiometer is able to record angles of twist up to $\pm 5^\circ$ (with a base of 1000 mm) and possesses an accuracy in measuring approximately 20 times greater than the mirror instruments. E. A. Raevskaya
Courtesy Referativnyi Zhurnal, USSR

1392. Thompson, J. M. T., Making of thin metal shells for model stress analysis, *J. Mech. Engng. Sci.* **2**, 2, 105-108, June 1960.

A method is outlined for the manufacture of small thin metal shells. Details are given of the production of a spherical copper shell of diameter 6 in., and wall-thickness 0.004 in. The shell is deposited electrolytically on a wax surface of the appropriate shape, and the wax is subsequently melted away. Arrangements of anode and cathode in the electrolysis process are examined for uniform thickness of metal deposit on surfaces of various shapes.

From author's summary

Material Test Techniques

(See Revs. 1245, 1251, 1290, 1342, 1377, 1382)

Properties of Engineering Materials

(See Revs. 1170, 1245, 1262, 1342, 1370, 1376, 1377, 1393, 1426)

Structures: Simple

(See also Revs. 1170, 1216, 1251, 1269, 1272, 1302, 1322, 1360, 1362, 1384, 1580)

Book—1393. Caracostea, A. D., edited by, Handbook for the computation of structures [Manual pentru calculul constructiilor], Bucharest, Editura Tehnica, 1959, 1965 pp. Lei 78.

This handbook is an up-to-date review of various problems met with in the computation and design of structures in Rumania.

It consists of thirteen sections. The first two sections present mathematical tables and results as well as data pertaining to loads and technical weights. Section III gives technological data concerning binding agents, mortars and concrete.

Section IV presents the most important results in the field of the strength of materials, obtained by using additional hypotheses on the deformation of bodies. The results obtained for beams on elastic medium and for thin-walled rods (V. Z. Vlasov's theory) are particularly worth noting. Section V deals with the plane problem of elasticity, giving especially results both for rectangular deep beams having eventually one or several sides extending to infinity, and for circular deep beams. Various methods of computation are given for periodical or local loads, the continuous and the single-span beams being examined. The problem of plane plates is discussed in section VI. Here plates with rectangular or circular boundaries are extensively studied and results pertaining to oblique and triangular plates are given. Considerations on the buckling of such plates with applications to metal structures are also included.

Section VII contains results obtained for thin shells. Following a general study of the stress and strain state in the membrane and bending theories, the stability of such plates is discussed. The results are applied to the computation of shells used for covering industrial halls, for reservoirs and for cooling towers. Elliptical and polygonal shells are particularly examined. Section VIII deals with the theory of structures. The most important topics pertaining to isostatic structures are presented, such as straight beams, latticed beams and arches. Hyperstatic structures such as curved beams and frames are also included. Certain particular problems are then examined, including Vierendeel beams, hyperstatic latticed beams and additional stresses in latticed beams.

Section IX treats briefly the problems of the pressure, strength and stability of the earth. Sections X, XI, XII and XIII deal with the computation of structural elements according to the respective material, i.e., wood, metal, masonery and reinforced concrete.

The whole subject is treated concisely and the results are generally accompanied by adequate theoretical studies.

P. P. Teodorescu, Roumania

1394. Wright, D. T., and Berwanger, C., Limit design of reinforced concrete beams, *Proc. Amer. Soc. Civ. Engrs.* **86**, ST 7 (*J. Struct. Div.*), 1–36, July 1960.

Authors present a method to predict the inelastic behavior of a two-span continuous reinforced-concrete beam, with a symmetrical concentric load in each span. Each individual span is similar to a beam fixed at one end and simply supported at the other. Moments and rotations at the critical sections were formulated both in the elastic and plastic range. Thus the ultimate capacity of the beam can be predicted based on its physical properties. Tests were made to verify the theory. P. C. Wang, USA

1395. Meier, H., Application of the Cross method for a beam of arbitrary distribution of inertia (in German), *Schiffstechnik* **7**, 36, 79–82, Apr. 1960.

Author shows the agreement of Cross method for beams and frames with variable moment of inertia. Although this method is simply based on Mohr's rules and different solutions have already been published in tabulated form, author's development of formulas for relative stiffness and carry-over factors forms a quick practical method for the calculation of the complicated problem.

The method is illustrated by a practical numerical problem.

E. Ben-Zvi, Israel

1396. Clarke, J. H., Method of assessing probable fire endurance of load-bearing columns, *J. Amer. Concr. Inst.* **31**, 12, 1223–1242, June 1960.

This paper deals with methods assessing probable fire endurance of load-bearing reinforced-concrete columns of square cross section, 15 in. × 15 in. to 12 in. × 12 in., subjected to fire test. Several factors influencing the endurance are analyzed, such as nature of aggregate employed, amount of free moisture, depth of the cover of reinforcement, etc. It is interesting that, for covers thinner than 1½ in., temperature of steel may exceed 1000°F after two hours of fire and contribution of reinforcement to load-carrying capacity is insignificant. Formulas for ultimate load are given and extensively discussed. Methods presented can be extended to smaller sizes of cross sections and to circular sections.

J. Nowinski, USA

1397. Grycz, J., On the transformations of a four-node structure by exchanging bars (in English), *Rozprawy Inz.* **8**, 1, 33–43, 1960.

A statically determinate four-node lattice is considered, the object being to find that arrangement of bars for which the material is utilized in the best way possible. The analysis is done for a six-bar lattice simply statically indeterminate. Three theorems are established, from which it follows that with four nodes and a given arrangement we can, depending on certain mathematical conditions, construct one, two or more lattices statically determinate and satisfying the minimum potential condition.

The physical sense of these conditions, on which depends the number of structures that can be obtained with four nodes and satisfy the minimum potential condition, is not explained. This sense is relatively clear: if four bars coincide with the funicular polygon drawn for the external forces, two minimum five-bar structures can be constructed. Each one will have one bar where the normal force is zero, Similarly if three, two or one bar can carry the entire load, two, three or four zero bars will appear in the lattice, respectively.

A numerical case is solved. Z. Kączkowski, Poland

1398. Fadle, J., The homogeneous, internally statically indeterminate parallel chord truss (in German), *Ing.-Arch.* **27**, 5, 314–325, 1960.

The Maxwell equations and the matrix force method are derived with special application to two-dimensional trusses. Expressions for temperature effects, horizontal and vertical deflections of the joints are worked out in a form most suitable to computer operations. Specialized application to temperature effects in homogeneous parallel chord trusses is given. The Maxwell equations can be written in terms of the diagonal angle, the number of fields and the coefficient of thermal expansion. The inverse of the coefficient matrix of the left hand side of these equations is derived as an exact, recurrent and also as an approximate, direct expression and in this manner a closed solution is presented. A worked out example is included. P. F. DeNeeve, Canada

1399. Belenya, E. I., Experimental investigation of the boundary conditions of composite steel frames under static loading (in Russian), *Dokladi Mezbvuz. Konferentsii po Isputaniem Sooruzb. Leningrad,* 1958, 120–128; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5546.

Tests were made on two-span six times statically undeterminable frames of an experimental framework for a single-storey commercial building. The cross-beams were grid-shaped, columnar-grid-shaped and continuous, supported on columnar concrete bases or on a reinforced plate. The span of the frames was 9 m, the height of the columns from the surface of the base to the bottom of the cross-beam, 5 m. The frames were tested by means of a vertical load applied at six joints of the upper collar of the collar beam of one of the spans or to the lower arm of one of the columns with a horizontal load at the level of the upper collar of the beam under the crane and with simultaneous vertical and horizontal loads. With a horizontal load or with a vertical load applied eccentrically to the

columns the boundary state for the deformations preceded the loss of carrying capacity and was observed as a regular feature at the moment of transition from the elastic to the elasto-plastic stage of operation. The loss of carrying capacity before the boundary state for the deformations was reached took place when applying loads to the grid-shaped collar beam. In addition the following special features of the work of the frames in an elasto-plastic state were established: (1) the breakdown of the linear relation of the displacements and stresses to the loads; (2) the loss of resistance to angular displacements of the foundations, this result being proposed as the basis of computations for the frames, taking into account the possibility of the appearance of plastic hinges at the level of the sole of the foundation; (3) the local loss of strength in the compressed flange and part of the wall in welded columns of double-I section when the usual ratios are observed for the thickness of the flanges and the walls to their width; (4) the loss of carrying capacity with loading less than that required by calculation for the formation of the full amount of plastic hinges. It is proposed to adopt for the boundary state of composite frames the moment at which loss of carrying capacity occurs in the first beam or at which the formation of the first plastic hinge takes place. A coefficient for the conditions of work of 0.5 is recommended for the beams in the grid-work of the columns, and for the mean sections of the cross-beam 0.75. G. M. Chuvikin

Courtesy Referativnyi Zhurnal, USSR

1400. Narets, L. K., Resolution of systems of canonical equations for structural mechanics (in Russian), *Trudi Tallinsk. Politekhn. In-ta* **A**, no. 121, 48 pp. + illus., 1957; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5649.

An investigation is made of a type of conversion of a system of equations which results in the separation of the system into independent groups of equations or into separate equations, each of which contains one unknown. In contrast to known investigations [*see*, for instance, I. M. Rabinovich, "Bases of the most recent methods for calculations of frame systems," Part 1. Moscow, Gosstroiizdat, 1933, chap. 7, para. 4] the methods of the theory of matrices are used, and the whole of the exposition is built up on the symbols of that theory. The procedure is given for the conversion of the matrices, which is required to solve the problem, and methods are described which simplify the calculations for the statically indeterminable systems with the aid of the small machine technique (the methods used by Gauss, Banakhevich, orthogonal conversions, etc.). Examples of the calculations for the frame and for lattice structures are examined, among them a ship's deck built on intersecting beams. I. K. Snitko

Courtesy Referativnyi Zhurnal, USSR

1401. Leicester, R., A resistance analogue of beams and arches, *J. Instn. Engrs., Australia* **32**, 4/5, 99–103, Apr./May 1960.

Paper shows similarity of equations governing bending of thin curved (or straight) beams, and current flow in resistance strips. Two examples dealing, respectively, with continuous beam and two-bay arch illustrate construction of an electric analog computer to simulate mechanical problems. In the analog computer described it is necessary to select redundant quantities by trial and error method or to use a "relaxation"-type technique to systematically reduce residuals. By latter method, author was able to "zero analogs of structures with two and three redundants within 20 and 100 seconds, respectively." Author discusses some practical problems such as shielding of analogs against stray electromagnetic fields, and accuracies obtainable with use of "sore-bought" resistors. B. Paul, USA

1402. Lebedinets, L. N., Investigation of and calculations for steel unarticulated arches in a boundary state (in Russian), *Sb. Leningrad In-ta Inzh. Zh.-d. Transport* no. 158, 91–110, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5541.

The usual assumptions are listed as the starting point of this investigation, namely: (1) the hypothesis of plane sections; (2) the hypothesis of an ideal elasto-plastic material; (3) the hypothesis of the boundary equilibrium in the boundary state, that is at the moment of the appearance of the last—the fourth—plastic hinge. The load is a single concentrated force. The computation is carried out by means of successive approximations, which were analytical and not graphoanalytical as stated by the author. The following investigations were carried out for arches of constant section: the influence of the longitudinal N force and the transverse Q force on the magnitude of the limit of load; the influence of changes in rigidity. It was established that (1) consideration of N in the computations was invariably imperative; (2) changes of N along the length of the arch could always be disregarded; (3) the influence of Q could always be disregarded; (4) the influence of changes in rigidity and outline of the axis was only appreciable in the case of rising arches. Quantitative characteristics for the various types of influence are given. A. A. Pikovskii

Courtesy Referativnyi Zhurnal, USSR

1403. Vedernikov, N. L., and Golovina, Z. M., Investigation on the deformation of round arches of small curvature (in Russian), *Trudi Rostovsk. N/D Inzh.-Stroit. In-ta* no. 6, 251–257, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5645.

The paper contains an experimental verification of the calculation formulas for a round girder of small curvature. The influence is investigated of compression of the axis and of intersecting forces on the magnitude of the components of the elastic forces and displacements. The experiments, which were carried out with flexible arches on steel bands loaded with concentrated forces, showed good concordance between the measured and the calculated deformations and also confirmed the necessity to take the compression of the axis into account when making calculations for slanting arches. A. B. Morgaevskii

Courtesy Referativnyi Zhurnal, USSR

1404. Slomyanskii, G. A., Temperature flexure of elastic elements, NASA TT F-34, 20 pp., May 1960.

Expression is developed for temperature correction to empirical load-deflection formula for elastic elements with temperature-dependent moduli. W. E. Jahsman, USA

Structures: Composite

(See also Revs. 1170, 1177, 1249, 1361, 1368, 1375, 1383, 1398, 1532, 1668)

1405. Duberg, J. E., Khachaturian, N., and Fradinger, R. E., Method for analysis of multibeam bridges, *Proc. Amer. Soc. Civ. Engrs.* **86**, ST 7 (*J. Struct. Div.*) 109–138, July 1960.

An analysis is performed for bridges with beam elements placed side by side and connected to each other along the span by hinges at the corners of the cross section at the level of top fiber. Influence line ordinates are computed for bending moments in beams with load moving along the transverse center line of the bridge. The reviewer knows of no practical use for such bridges. K.-H. Chu, USA

1406. Uchida, I., Solution of the spandrel-brace-type Vierendeel bridges, *Mem. Fac. Engng., Kyushu Univ.* **19**, 3, 72–114, Mar. 1960.

Paper presents equations needed for solving for the stresses and deflections of a Vierendeel truss of the spandrel-brace type. In one set of equations the expansion and contraction of all truss members are considered; it can thus be used for determining tem-

perature effect. In another set of equations the expansion and contraction of members are not considered. No numerical results are presented, and no discussion is given as to what influence the expansion and contraction may have on the stresses and deflections.

Y.-Y. Yu, USA

1407. Harrenstien, H. P., and Alsmeyer, W. C., Structural behavior of a plate resembling a constant thickness bridge abutment wingwall, Iowa State College, Iowa Engng. Exp. Sta. Bull. 182, 22 pp., Jan. 1959.

The method of finite differences was used to solve the Lagrange plate equation written for a plate of the wingwall type.

Solutions were obtained corresponding to six different grid spacings.

The various solutions were compared with experimental results obtained by testing a ½-in. aluminum plate which was considered a model of a typical wingwall.

This comparison indicated that satisfactory predictions of structural behavior may be made from the set of equations corresponding to the 6 × 6 grid (36 simultaneous equations).

From authors' summary

1408. Hulsbos, C. L., and McCasland, W. H., Moment contours for bridge abutment wingwalls of constant thickness, Iowa State College, Iowa Engng. Exp. Sta. Bull. 183, 27 pp., Feb. 1959.

Further experimental verification of the finite difference numerical method was made by tests on a wingwall model of a different size proportion than that investigated in part two [see preceding review] of the project. This investigation gave substantial evidence to the accuracy and reliability of the numerical method.

The development of bending moment contours by use of the finite difference method enables the calculation of moment at any point in the wingwalls. Contours were developed for maximum principal bending moment and for moments in the x and y directions. The directions of the maximum principal moments were found and presented in the form of moment trajectories. The equation for the moment calculation to be used in connection with the contours is given as

$$M = \frac{C_{p_f} a^2}{10^4}$$

where

C = coefficient from the moment contours
p_f = normal soil pressure at the fixed-fixed corner in psf
a = base length of wingwall in feet

The accuracy of the bending moments determined from the contours is approximately ±8% along the fixed boundaries and ±15% in the central region where the error, though large in percentage, is small in magnitude.

An investigation was made of the footing rotation with its resulting effects on the moments in the wingwall. It was concluded that a factor of 1.2 to be used in the previously mentioned moment equation was satisfactory to cover the moment increase along the breastwall boundary and in the central region of the wingwall. The equation for these moments will be

$$M = \frac{1.2\, C_{p_f} a^2}{10^4}$$

From authors' summary

1409. Hulsbos, C. L., and Lee, T., Moment analysis for bridge abutment wingwalls of variable thickness, Iowa State College, Iowa Engng. Exp. Sta. Bull. 184, 36 pp., May 1959.

A finite difference method for the solution of moments in a wingwall of variable thickness is presented herein. A differential equation which governs the deflections of a wingwall was derived by the application of the plate theory. Boundary conditions in the form of differential equations were derived on the assumption of two fixed edges and two free edges. These differential equations were transformed into a set of simultaneous linear algebraic equations in terms of unknown deflections at grid intersections by the application of the finite difference procedure. The deflections at grid intersections obtained from the solution of these equations were used to calculate the moments in the wingwall. An experimental investigation was conducted on an aluminum model wingwall. The resulting deflections and moments compared favorably with the theoretical results obtained from the finite difference solution. It is concluded therefore that, by the application of the finite difference method as developed in this investigation, the moments in a bridge abutment wingwall due to lateral soil pressure can be calculated with a reasonable accuracy.

From authors' summary

1410. Weil, N. A., and Murphy, J. J., Design and analysis of welded pressure-vessel skirt supports, *ASME Trans.* **82** B (*J. Engng. Indust.*), 1, 1–14, Feb. 1960.

Paper presents a method for determining stresses in high-temperature vessels and their skirt attachment welds. The analysis is applicable not only to skirt supports, but also to the stress analysis of composite vessels with a single, internal head. Authors believe that the moments resulting from axial thermal gradients are the major cause of skirt-weld cracking.

C. B. Matthews, USA

1411. Numata, E., Longitudinal bending and torsional moments acting on a ship model at oblique headings to waves, *J. Ship Res.* **4,** 1, 35–48, June 1960.

Tests were conducted with a model of a T2-SE-A1 tanker running at oblique headings to waves to measure torsional moments and vertical and lateral components of longitudinal bending moments at the midship section. Trends of moments with speed and wave direction in regular long-crested waves of various lengths are presented. Statistical characteristics of moment responses at one speed and heading in irregular long-crested waves are given. The relative importance of lateral bending is explored by using model results to predict longitudinal bending stresses in a T-2 tanker deck caused by the combined action of vertical and lateral moments, and it is shown that lateral bending can contribute significantly to these stresses.

From author's summary

1412. Grzedzielski, A. L. M., Note on some applications of the matrix force method of structural analysis, *J. Roy. Aero. Soc.* **64,** 594, 354–357 (Tech. Notes), June 1960.

Paper discusses a modification of the matrix force method as proposed by Argyris [Aero. Res. Counc. Lond. Rep. Mem. 3034 1957; AMR **11**(1958), Rev. 3994]. Author did not like the concept of the initial strain originally introduced in connection with thermal stress and cut-out techniques; he proposes new procedures for analyzing those cases in which the flexibility matrix is not only diagonal. Author states that the modification is essential for numerical analysis of low-aspect-ratio swept wings.

Y. C. Fung, USA

1413. Santini, P., Fuselage structures of noncircular section (in Italian), *Monogr. Scient. Aero.* no. 5, 31 pp., June 1959.

Linear deformation analysis of a monocoque fuselage-type structure with stringers and rigid section rings. Symmetric sections about one axis, no taper. Loading conditions (a) symmetric, (b) antisymmetric are considered separately. Deformation is written as superposition of (a) rigid section displacements and (b) distortion. The "distortion function" is determined by means of Broglio's "balance method."

C. Riparbelli, USA

1414. Kalandiya, A. I., A direct method for the solution of the theory for a wing and its application in the theory of elasticity (in Russian), *Matem. Sb.* **42**, 2, 249–272, 1957; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5411.

The paper starts with an investigation of the singular integro-differential equation for the theory of a wing of finite span

$$\frac{\Gamma(x)}{B(x)} = \frac{1}{2\pi}\int_{-1}^{+1} \frac{\Gamma'(t)}{t-x}\,dt = f(x) \qquad [1]$$

where $B(x)$ and $f(x)$ are assigned functions which on cutting $[-1, +1]$ satisfy the determined conditions, while $\Gamma(x)$ is the function being sought, satisfying in its turn some (stated) conditions. This is followed by a brief description of Multhopp method [H. Multhopp, *Luftfabrtforscbung* **15**, no. 4, 1938], the gist of which can be given as follows: by substitution of $x = \cos\theta$ ($t = \cos\tau$) Eq. [1] assumes the form of

$$\frac{\Gamma(\vartheta)}{B(\vartheta)} + \frac{1}{2\pi}\int_0^{\pi} \frac{\Gamma'(\tau)}{\cos\tau - \cos\vartheta}\,d\tau = f(\vartheta) \qquad [2]$$

The number for n is fixed; the following points are taken as units

$$x_k = \cos\vartheta_k,\ \vartheta_k = \frac{k\pi}{n+1} \qquad (k = 1, 2, \ldots, n)$$

and a trigonometrical interpolational Langrange polynome is derived

$$L[\Gamma; x] = \sum_{k=1}^{n} (-1)^{k+1}\,\Gamma(x_k)\,\frac{\sin(n+1)\vartheta\,\sin\vartheta_k}{\cos\vartheta - \cos\vartheta_k} \qquad [3]$$

There is a formula in mechanical quadratics for the singular integral

$$\frac{1}{2\pi}\int_0^{\pi} \frac{\Gamma'(\tau)}{\cos\tau - \cos\vartheta}\,d\tau = \frac{1}{n+1}\sum_{k=1}^{n}\Gamma(x_k)\times$$

$$\times \sum_{m=1}^{n} \frac{m\sin m\vartheta_k \sin m\vartheta}{\sin\vartheta} \qquad [4]$$

With the help of this formula and by giving argument ϑ progressively the values of $\vartheta_k (k = 1, 2, \ldots, n)$, a system of linear equations is obtained for the determination of the approximate values Γ_k of the sought function at points ϑ_k

$$b_\nu \overline{\Gamma}_\nu = f_\nu + \sum_{k=1}^{n} b_{\nu k}\,\overline{\Gamma}_k \qquad (\nu = 1, 2, \ldots, n) \qquad [5]$$

where

$$b_\nu = \frac{1}{B_\nu} + \frac{n+1}{4\sin\vartheta_\nu}$$

$$b_{\nu k} = \frac{1}{4(n+1)\sin\vartheta_\nu}\left[\operatorname{ctg}^2\frac{\vartheta_k - \vartheta_\nu}{2} - \operatorname{ctg}^2\frac{\vartheta_k + \vartheta_\nu}{2}\right]$$

$$\text{with } |\nu - k| = 1, 3, \ldots$$

$$b_{\nu k} = 0 \quad \text{with} \quad |\nu - k| = 2, 4, \ldots$$

By substituting solution Γ_k of system [5] in [3] in place of $\Gamma(x_k)$ an approximate solution for equation [1] is found in the form

$$\Gamma_n(x) = \frac{1}{n+1}\sum_{k=1}^{n}(-1)^{k+1}\,\overline{\Gamma}_k\,\frac{\sin(n+1)\vartheta\,\sin\vartheta_k}{\cos\vartheta - \cos\vartheta_k} \qquad [6]$$

Then, by means of the method of functional analysis, using the determined conditions, the fact is disclosed that the process of successive approximations for the solution of system [5] is convergent. The paper continues by putting forward some propositions (used later on in the paper) from the theory of the uniform approximation of the functions, assigned on the cutting. It is established by means of the methods of functional analysis, with some conditional reservations, that the sequence of $\Gamma_n(x)$ merges evenly with the solution of Eq. [1].

The study of the mixed problem in the plane theory of elasticity is then taken up. An investigation is made of an elastic medium filling the interior of a unit circle; it is assumed that boundary L of the area is divided into two parts L_1 and L_2 by points $\zeta = \pm 1$. The elastic equilibrium of the circle is sought with the following boundary conditions in force

$$T_0 = N_0 = 0 \ \text{ on } \ L_1$$

$$T_0 = 0, \quad \nu_0 = g(\sigma) + c \ \text{ on } \ L_2$$

where T_0 and N_0 are the tangential and normal components of the external pressure, ν_0 is the normal displacement, $g(\sigma)$ is the assigned function satisfying the determined conditions, while c is a constant subject to determination. By deriving the function of a complex variable $\Phi(\zeta)$ and $\Psi(\zeta)$ and by using the N. I. Muskhelishvili method the author of the solution of the problem set brings about the finding of function $N(x)$ which satisfies the equation

$$\frac{N(x)}{B(x)} - \frac{1}{2\pi}\int_{-1}^{+1}\frac{N'(t)}{t-x}\,dt = f(x) - \frac{\gamma c}{x^2+1} \qquad [7]$$

of the form of [1] where

$$N(x) \equiv N_0\left(i\,\frac{x-i}{x+i}\right), \qquad B(x) = -\frac{x^2+1}{k}$$

$$f(x) = -\frac{\gamma}{x^2+1}\left[g(\sigma) + \frac{d^2 g(\sigma)}{dx^2}\right]$$

$$\left(k = \frac{\kappa-1}{\kappa+1}, \qquad \gamma = \frac{4\mu}{\kappa+1}, \qquad \sigma = e^{i\alpha}\right)$$

The constant c is determined from the condition

$$\int_{-1}^{+1}\frac{(1-t^2)\,N(t)}{(1+t^2)^2}\,dt = 0$$

The determination of the functions $\Phi(\zeta)$ and $\Psi(\zeta)$ is then carried out by means of the correlations

$$\Phi(\zeta) = \frac{1}{2\pi i}\int_{L_2}\frac{N_0(\sigma)}{\sigma-\zeta}\,d\sigma - \frac{1}{4\pi i}\int_{L_2}\frac{N_0(\sigma)}{\sigma}\,d\sigma$$

$$\Phi'(\zeta) + \zeta\,\Psi(\zeta) = 0, \qquad |\zeta| \leqslant 1$$

An approximate solution for equation [7] is found by the method already described in this paper

$$\overline{N}(x) = \frac{2\sqrt{1-x^2}}{n+1}\sum_{m=1}^{\frac{n}{2}} a_m \sum_{k=0}^{\frac{m}{2}}(-1)^k\,\frac{m-2k}{m-k}\,c_{m-k}^{k}\,(2x)^{m-2k-1}$$

$$a_m = \sum_{k=1}^{n}\overline{N}_k \sin m\vartheta_k$$

Here $\overline{N}_k$ is the solution corresponding to the linear system. Then follows a determination for the approximate values for the function $\Phi(\zeta)$. A case is examined as an example when

$$g(\sigma) = -\frac{1}{4}\left(\sigma - \frac{1}{\sigma}\right)^2,$$

and where $n = 7$ and $n = 15$ the calculations are carried out to the end.

A. K. Rukhadze

Courtesy Referativnyi Zhurnal, USSR

1415. Sokol'skii, M. M., Deformations in hydrotechnical installations, their nature and methods of calculation (in Russian), *Izv. Vyssh. Uchebn. Zavedenii. Geod. i Aerofotos'emka* no. 4, 5–19, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5603.

The paper makes available: (1) factual data for the settlement of large installations; observations recorded a mean settlement up to 15 cm, up to 70% of these after the period of construction; (2) graphs for changes in settlement and the divergence with lapse of time from the vertical of installations, situated in the Kuibyshevsk hydro network. No theoretical corroboration for the observations is given. Some information is furnished on the methods used and the results of the solutions for problems in the theory of elasticity in conditions of even pressure on a semi-plane (plane deformation) and a semi-space.

I. K. Snitko

Courtesy Referativnyi Zhurnal, USSR

Machine Elements and Machine Design

(See also Revs. 1170, 1198, 1317, 1527)

1416. Pacejka, H. B., Theoretical treatise on the behaviour of air springs, *Ingenieur* **72,** 10, 61–74, Mar. 1960.

The behavior of air springs is investigated theoretically on the basis of the assumptions that the axial cross section of the reinforced diaphragm has a constant developed length and as to the free part, a circular shape. Both the axial and the lateral behavior of air springs are examined; more specifically, the axial stiffness of a general type of air spring and the load-deflection curves of some particularly shaped air springs of the bellow and diaphragm type are given as well as the lateral stability and stiffness of air springs with no more then two convolutions.

From author's summary

1417. Ludwig, L. P., Engagement characteristics of wet-type clutches, *SAE Trans.* **67,** 391–400, 1959.

Paper attempts to present a theoretical approach and some related experimental tests on the heat-generation problem in a clutch during engagement, and ends with a discussion. The mathematical analysis is rather elementary and does not contribute much to the existing knowledge of clutch mechanics. The mean radius derivation under conditions of constant pressure and constant wear can be found in many textbooks. The analysis of slip torque curve of engagement period, under assumption of constant slip torque, is a straightforward case of mechanics of a system on which a constant torque or a force is acting. The set of equations is therefore, in the opinion of the reviewer, more window dressing rather than essential for discussions. The graphs and corresponding verbal statements of theoretical conclusions are enough to carry the point of view of the author. Nevertheless, the test results described are interesting.

B. M. Belgaumkar, India

1418. Davydov, V. I., Investigation of bending combined with tension (in Russian), Inzhenern. Metodyi Rascheta Tekhnol. Protsessov Obrabotki Metallov Davieniem, Moscow-Leningrad, Mashgiz, 1957, 149–166; *Ref. Zh. Mekh.* no. 3, 1959, Rev. 3028.

The paper consists of a theoretical investigation of bending with tension, principally for the practically important large radii of curvature. It is demonstrated that the magnitude of the residual elastic deformation—the angle of spring—will be a minimum value in the case where the magnitude of the tensioning stresses caused by supplementary tensioning forces or by supplementary deformations is close to the value of the limit of yield of the material being bent; this applies to cases of bending with tension produced by the application of supplementary tensioning forces and by the application of additional deformation of the material. It is noted that for these two forms of bending the magnitude of the angle of spring depends also on the shape of the material's section through the magnitude of the brought-in moment of resistance. For the case of bending of a bar, the minimum magnitude of the angle of spring will be found when the section of the bar is chosen in a way that will ensure that during the bending the whole section of the material shall be in the elasto-plastic zone.

S. V. Zhuravlev

Courtesy Referativnyi Zhurnal, USSR

Fastening and Joining Methods

(See also Rev. 1410)

1419. Zaborov, V. I., Calculation for the joints of vertical panels in houses having no framework (in Russian), Questions relating to calculations for the construction of domestic and civil-purpose houses with prefabricated elements, Moscow, Gosstroiizdat, 1958, 172–190; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5727.

Author analyzes the construction of a wall built out of separate rectangluar panels, as representing a combination of stiff disks united along the vertical seams by elastic links under the action of both tension (compression) and shear; in the horizontal plane the seams are assumed to have a rigid joint between the panels. The possibility of translating a similar schematic plan into constructional practice is not touched on. A case is investigated in which such a system is supported on an elastic soil base with various characteristics along the length of the wall; the solution of the problem is brought to two systems of linear algebraical equations for the determination of the forces in the seams. Graphs are drawn to express the relationship between the forces in the seams, the settling and displacements in the joints and the elastic characteristics of the foundations and the number of stories in the building.

A. S. Kalmanok

Courtesy Referativnyi Zhurnal, USSR

1420. Navrotskii, D. I., Investigation of the strength of welded joints when residual stresses and local plastic deformations are acting separately (in Russian), *Trudi Leningrad Politekhn. In-ta* no. 189, 43–50, 1957; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5771.

An investigation for the fatigue strength of steel was carried out with test samples made up from three bands welded together in cruciform shape. In the dangerous section of the sample an uneven distribution of stresses was obtained with considerable stress concentration. The residual stresses were produced by peening the edges of the sample or by strongly heating them. The magnitude of the residual stresses was verified by cutting out separate portions and fastening strain gages on them. The experiment showed that positive residual stress (which was the mean stress of the cycle) lowers the fatigue strength, while negative residual stress increases it to approximately the same extent. Approximate formulas are proposed for the evaluation of the degree of influence of the residual stresses.

S. V. Boyarshinov

Courtesy Referativnyi Zhurnal, USSR

1421. Navrotskii, D. I., Determination of the tangential stresses in a welded butt joint (in Russian), *Trudi Leningrad Politekhn. In-ta* no. 189, 58–67, 1957; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5772.

The author evolves a schematic plan for the form of strengthening of butt joints. The actual profile of the seam in calculating its strength is replaced by a simplified one with a rectangular or inclined protrusion. An investigation is then carried out on the formation of tangential stresses in the seam, distributed in the boundary zone between the basic plate and the reinforcement. Au-

thor studied the distribution of the tangential stresses with different forms of protrusions in relation to their initial height and length. The determination of the concentration coefficients for the tangential stresses in functions of normal stresses evenly distributed over the plate was carried out by an analytical method in accordance with formulas proposed by the author.

G. A. Nikolaev
Courtesy Referativnyi Zhurnal, USSR

Rheology

(See also Rev. 1690)

Book—1422. Kuhn, A., editor, Manual of colloid chemistry [Kolloidchemisches Taschenbuch], 5th ed., Leipzig, Geest & Portig, 1960, 555 pp. Price DM. 28 ($6.80).

This is a comprehensive, composite work of 19 authors from nine countries, each a specialist in various branches of colloid chemistry and physics. The material is organized under the following chapters: A. System of colloids (two-phase systems, homogeneous colloids, macromolecular compounds, colloids considered as discontinuous systems); B. Generation of colloids (theoretical relationships, principles of generation, measuring methods, surface active colloids); C. Kinetics of colloids (molecular motion, sedimentation, osmosis and diffusion, dialysis, ultrafiltration, electrodialysis, apparatus for various applications, coagulation, dispersoid analysis, particle-size determination); D. Optical properties of colloid systems (Tyndall-phenomenon, specific absorption, anisotropy, optical activity, x-ray analysis, morphology and structure); E. Boundary-surface phenomena (surface tension, monomolecular layers, adsorption, electrical phenomena); F. Special systems (aerosols and their production); G. Molecular colloids (macromolecules, rheology of disperse systems, measuring methods and instruments, gels, swelling, theories, experimental methods).

Each chapter is subdivided into numerous subjects, and extensive literature is listed, collected from many countries. A very complete name index and subject index is provided. This is an excellent reference work condensing in a handy volume, a great deal of information pertaining to a field of ever-growing extent and importance.

K. J. DeJuhasz, USA

1423. Korol', I. P., Study of viscous flow of rubbers: Part I, Method of constant deformation; Part II, Flow of rubbers during constant deformation; Part III, Flow of concentrated solutions of some rubbers, *Soviet Phys.-Tecb. Phys.* **4,** 4, 420–437, Oct. 1959. (Translation of *Zb. Tekb. Fiz., USSR* **29,** 4, 471–491, Apr. 1959 by American Institute of Physics, New York, N. Y.)

Part I describes the application of the mathematics of a combined Maxwell and Kelvin spring-dashpot system to the flow of different molecular weight grades of polyisobutylene. Molecular weights ranged from 0.2 to 3.0×10^6 and the temperature from 40°C to 120°C. Experimentally, a method of constant deformation was employed using small cylindrical samples. The relative values of viscous deformation and highly elastic deformation were determined as a function of time and recovery. Residual, irreversible deformation is an approximately linear function of log time. Variations in molecular weight, temperature, and degree of initial displacement result in equi-distant lateral displacement of the irreversible deformation-log time curves. A characteristic time has been found which governs the flow under a given set of conditions. It is concluded that the transformation of highly elastic deformation into viscous deformation is the governing factor in the relaxation process.

Part II describes the application of the above technique to five synthetic and natural rubbers. The log of the characteristic time was found to be a linear function of the log molecular weight, temperature, and degree of compression. The kind of rubber may be identified by empirical constants, one of which is related to the structure of the rubber polymer molecules. Author concludes that experiments in flow of solid rubber rather than solutions are necessary to define a "molecular coefficient".

Part III is concerned with the data given in Part II which showed that the structural parameters for the flow of infinitely diluted solutions and for solid rubbers are extremely different. Above a certain concentration, however, these differences may be resolved and the author concludes that above a minimum concentration a study of viscous flow of concentrated solutions may be easier, experimentally, and lead to the same conclusions as studies of pure rubbers.

R. H. Carey, USA

1424. Safronchik, A. I., Non-steady flow of a visco-plastic material between parallel walls, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **23,** 5, 1314–1327, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

An incompressible viscoplastic material is considered, one which behaves as an elastic solid below a critical shear stress and as a viscous fluid above it. The problem treated is that of the unsteady flow of this material between two infinite parallel planes, and a coordinate system with the xz plane as the plane of symmetry of the flow and the y-axis perpendicular to it is adopted. The only non-zero velocity component is $v_x = v_x(y, t)$ and the only non-zero pressure gradient is $\partial p/\partial x = P(t)$.

While the material next to the wall behaves as a viscous fluid, there is a core $|y| \leqslant y_0(t)$ which behaves as an elastic body. The determination of this boundary between the elastic and viscous regimes leads to a Stefan problem, that is, a moving boundary problem for the heat equation. Author treats this by the method of Kolodner, deriving an integral equation for $y_0(t)$. For the special case in which $P(t)$ is a step function, an approximate solution for the short-time behavior of $y_0(t)$ and of the velocity of the core is obtained.

J. J. Weiner, USA

1425. Pindera, J., Investigation of certain rheological properties of some polyester resins from the point of view of photoelasticity, Part I (in English), *Rozprawy Inz.* **7,** 3, 363–411, 1959.

In the present state of development of photoelastic methods particular attention should be paid to the construction of models and, in particular, to the investigation of the properties of photoelastic materials subjected to long-duration loads or, in other words, to the creep tests.

The methods now in use consist in bending beams or stretching prismatic bars, with long-duration constant loads. The present paper describes tests of photoelastic creep. The disadvantages of the test methods now in use are pointed out and a new method is proposed consisting of using a model in the form of a wedge of small taper subjected to tension with a constant axial force which gives practically a one-directional state of stress. The test technique is described in detail.

Next, using test pieces of another very simple model, the influence of alternating loads on the phenomenon of photoelastic creep is investigated, this investigation being generalized then to the relaxation phenomenon.

Using data avialable from the literature the structure and the basic properties of the CR-39, VP-1527 polyester resins and the P-6 Palatal are discussed.

Author also proposes a test program for creep and relaxation for static and alternating loads such as to enable the results to be interpreted from the point of view of the internal structure and the rheologic properties, thus constituting in some sense an introduction to further tests by means of photoelastic vibro-creep methods.

J. Naleszkiewicz, Poland

1426. Shvidkovskii, E. G., The viscous properties of liquefied metals (in Russian), The hydrodynamics of molten metals, Moscow, Izd-vo Akad. Nauk SSSR, 1958, 45–55; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5342.

A detailed description is given of a quasi-crystalline model of the structure of metallic liquids, based on physical experiments. As shown in these experiments, when substances are liquefied, and are known to have a monocrystalline or a polycrystalline structure, the crystalline lattice is only partially disrupted; in the liquid state of the substance near individual atoms the coordination in the distribution of the atoms is conserved, analogous to the coordination of the atoms in the crystals. At a distance from these regions, named "quasi-crystalline", the structure of the liquid acquires an unregulated character in regard to the arrangement of the atoms. The predominance of one or another character in the structure of the substance is determined by the temperature. Inasmuch as the viscosity of the liquids is determined by their structure, generally speaking, it is possible to advance an explanation for a number of their viscous properties, based on the assigned model of the liquids. Thus, for instance, viscosity hysteresis in metallic alloys can be explained by the variable probability of transitions from the liquid state to the solid and from the solid to the liquid, linked up with the preeminent concentration in the disorderly zone of the atoms of one of the components of the alloy. Questions are touched upon regarding the flow resistance and heat exchange from the liquid to the wall during the motion of the liquefied metals along tubes. Here the assumption is made that the motion of these metals along the tubes is analogous to that of the motion of water.

A. I. Golubev

Courtesy Referativnyi Zhurnal, USSR

1427. Nikolaev, V. A., and Samarina, I. A., The dependence of the elasto-viscous properties of dough on the basic elements of its structure (in Russian), The bakery and pastry cooking industry no. 3, 3–8, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5850.

Results are given of tests carried out on the elastoplastic properties of the individual components entering into the composition of flour dough (albumens, gluten, concentrated suspensions and some thickeners). An investigation took place regarding the influence of the electrolytes contained in the water used for mixing the dough on the physical properties of the gluten washed out of the dough. The experiments showed that the deformation of the gluten up to a certain point can be described by a model of Kelvin's body, while the starch paste approximates that of Maxwell's body. The mechanical properties of the humidified bran also show that this structure can be likened to a model of Maxwell's body.

V. A. Sveshnikova

Courtesy Referativnyi Zhurnal, USSR

1428. Crussard, C., New rheological model of flow (in French), *Rev. Métall.* **55**, 4, 375–382, Apr. 1958.

A linear viscoelastic model is proposed for the representation of creep-curves which consists of a series of spring-connected viscous dash-pots of equal linear viscosity representing interacting "slip centers." The associated primary creep relation is a sum of exponentials which can be made to fit observations quite well and can also be represented by a single power function over a limited range. However, the linearity (with respect to stress) of this model makes its validity rather dubious beyond the range of very low stresses.

A. M. Freudenthal, USA

Hydraulics

(See also Revs. 1367, 1373, 1415, 1426, 1441, 1502, 1503, 1552, 1556, 1702, 1709)

Book—1429. Rauch, A., Handbook of hydraulic power stations [Taschenbuch Wasserkraftanlagen], Stuttgart, Franckh'sche Verlagshandlung, 1959, 429 pp. DM 39.

1430. Chavtoraev, A. I., et al., Pressure losses in gratings (in Russian), *Gidrotekh. Stroit.* **27**, 5, 44–47, May 1958.

The usefulness and proper use of various empirical relations for different types of gratings, grids and screens, used in hydroelectric facilities ahead of turbines, are discussed.

A. G. Fabula, USA

1431. Filippovich, I. V., Ejection through the open turbine chamber at an hydro-electric station of low power (in Russian), Questions on the operation of hydro-electric stations in the BessSR, Minsk, 1958, 63–84; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5101.

The ejection into the lower water is investigated in the case of a hydro-station with an open turbine chamber and a spillway orifice in a wall directed towards the lower water. The author, adopting the usual assumptions, derives an equation for the increment of the quantity of motion for the conditions under investigation. The laboratory verification of the calculation relationships was carried out on a model of the installation consisting of two plant units, on the scale 1:20, in a tank 60 cm wide, with turbine replacements of corresponding resistances. The author comes to the following conclusions: (1) The adoption of ejection through the turbine chambers permits the passage through the hydro-electric station of up to 25% of the flood waters of 1% guarantee, thus increasing the pressure (up to 30%) and discharge (up to 15%) of the turbine. (2) In order to avoid too great an increase in the loss of pressure in the turbine chamber the relation of the discharge of the spillway to the turbine's should be not more than 2 to 2.5. With values for this relation exceeding 5 the piezometrical effect of ejection is less than the loss of pressure in the turbine chamber. (3) The level of the crest of the spillway should not be lower than the upper casing of the turbine. The location of the turbine in front of the spillway reduces the coefficient of discharge of the spillway by 5 to 6%. (4) The proposed calculation relations are confirmed sufficiently well by the experiments on the model.

S. A. Egorov

Courtesy Referativnyi Zhurnal, USSR

1432. Karas, K., Stationary laminar flow through canals of rectangular cross sections in the case of constant pressure or static pressure variation (in German), *ZAMM* **39**, 3/4, 146–160, Mar./Apr. 1959.

Author presents an analogy between the title problem and the form of a uniformly loaded membrane of rectangular cross section. The solution is considered in form of series, the rapid convergence of which permits the construction of velocity relief maps by an analytic-graphical procedure.

Three numerical examples are given.

D. Gh. Ionescu, Roumania

1433. Chapman, T. G., Capillary effects in a two-dimensional ground-water flow system, *Géotechnique, Lond.* **10**, 2, 55–61, June 1960.

Effect of capillary rise on two-dimensional flow through a bank with vertical faces is formulated and checked by electrical analogy. An increase in capillary rise not only raises the free surface near the downstream face but lowers it near the upstream face. Water flows into the capillary zone near the point of entry and crosses the free surface again near the downstream face.

J. C. Geyer, USA

1434. Fomichev, M. S., Effect of submergence depth on the hydrodynamics of lower water level (in Russian), *Gidrotekh. Stroit.*

29, 6, 35–41, June 1959.

Although paper is interesting from the theoretical point of view, today it has only scant interest for the practicing engineer. Article deals with some dynamical phenomena in the discharge of small spillways and author gives formulas for the relation between certain hydraulic and dynamic characteristics of the flow. These results were gotten through tests, but unfortunately paper gives only brief information about the devices employed and on the sampling of the test results that made it possible to obtain those formulas.

J. L. Sanchez Bribiesca, Mexico

1435. Ibad-Zade, Yu. A., Hydraulic elements of channel straightening (in Russian), *Gidrotekh. Stroit.* **29**, 7, 49–51, July 1959.

Using variational methods author obtains a simple criterion to design the transversal section for straightening of rivers on the hypothesis that the best section is that of maximum hydraulic ratio. Article is only a brief report on those items and, in the absence of numerical examples, it is quite difficult to know the true usefulness of the formulas.

Author gives four references of his own works in which perhaps the reader can get an ample explanation of the presented method.

J. L. Sanchez Bribiesca, Mexico

1436. Brand, R. S., The collapse of a spherical cavity in a compressible liquid, Brown Univ., Div. Appl. Math. TR 34 (Contract Nonr 562(07) (NR-062-179), 45 pp., Apr. 1960.

The collapse of a vacuum sphere in a fluid is treated with the following assumptions: At time $t = 0$, an infinite expanse of liquid is at rest at pressure p_0 above the vacuum of the spherical cavity of radius r_{w0}. The equation of state is $(p + B)/(p_0 + B) = (\rho/\rho_0)^n$, where for the numerical calculation $n = 7$, $B = 3000$ atm. The cavity walls remain spherical during collapse. Viscosity and surface tension are neglected.

First, the problem is solved numerically by the method of characteristics for an initial pressure p_0 corresponding to a ratio of velocity of sound at the cavity wall to initial velocity of sound c_0 of 0.97. Among other quantities, the velocity of sound, c, is plotted as a function of the radius at the time when the cavity wall radius is 0.0739 of the initial cavity radius. The maximum velocity of sound is reached ($c = 2.26c_0$) very near the cavity wall. This value corresponds to an extremely high pressure. The velocity of the cavity wall at this time is $1.52c_0$.

Second, an analytical solution of the Langrangian equation of motion by expanding in powers of essentially the density changes is presented. While there is agreement with the above calculations regarding the motion of the cavity walls as a function of time (it takes about $8.5r_{w0}/c_0$ for the bubble to collapse), no high pressures are found.

H. P. Eichenberger, USA

1437. Baumberg, I. D., Controlling discharge of liquids by the bubble method (in Russian), *Trudi Gruz. Politekhn. In-ta* no. 3(60), 36–43, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5369.

The essence of the bubble method for measuring the discharge of liquids lies in the automatic division of the liquid flowing through a glass tube into separate portions of equal volume and a count being taken of the number of portions flowing through in a unit of time. The division of the liquid is effected by using air bubbles (stoppers) introduced into the flow by means of a fine metallic tube having a diameter of 0.4–0.5 mm. The constancy of the portions of liquid sealed off by two neighboring air stoppers is ensured by the automatic introduction of the next air stopper at a strictly determined moment, established by passing the previous air bubble through a photo-relay located at some distance away from the point of introduction of air along the flow of the liquid. Altering this distance varies the volume of the portions. The calculation of the number of portions passing through in a given unit of time is carried out by an electromechanical counting device. The measuring apparatus consists of a tube for the inlet of air, a valve, for the inlet of air, an electromagnet, controlling the operation of the valve, an amplifier and a photo-relay consisting of a photo-element and an illuminator. The work of the photo-relay is dependent on the deflection of a beam of light falling on the photo-element during the transit of an air bubble.

The proposed method of measurement is applicable both to clear and opaque liquids for the measurement of small discharges (up to 9/hr) flowing through glass tubes 2–4 mm in diameter at a velocity of 20 cm/sec. The possibility is mentioned of employing a similar device for the measurement of the velocity of a flow. For this purpose the circumstance is made use of that, when the bubble passes across the photo-relay the luminosity of the photo-element does not change suddenly but gradually in relation to the velocity of the flow. That being the case, the magnitude of the electromotive force on the secondary winding of the outlet transformer will be proportional to the velocity of the flow.

N. M. Ivanov

Courtesy Referativnyi Zhurnal, USSR

Incompressible Flow

(See also Revs. 1432, 1469, 1476, 1477, 1520, 1521, 1523, 1547, 1548, 1560, 1566, 1575, 1602, 1606, 1608, 1615, 1637, 1643, 1689, 1717)

1438. Hess, J. L., Calculation of potential flow about bodies of revolution having axes perpendicular to the free stream direction, Douglas Aircr. Co. Rep. ES 29812, 96 pp., Oct. 1960.

This report describes a general method of calculating, with the aid of an electronic computer, the potential flow about arbitrary bodies of revolution whose axes are perpendicular to the free-stream direction. When combined with the solution for the axisymmetric flow about these bodies, this method makes it possible to calculate the pressure distribution on any body of revolution at angle of attack forward of any separated region of the flow and also to calculate the flow at points off the body surface. The method is derived, checked against the analytic solution for ellipsoids of revolution, and compared with experimental data for a variety of bodies, including inlets with centerbodies. The calculated flow about certain other bodies is also presented to exhibit certain properties of this type of flow.

From author's summary

1439. Lambourne, N. C., and Bryer, D. W., Some measurements in the vortex flow generated by a sharp leading edge having 65 degree sweep, Aero. Res. Counc. Lond. Curr. Pap. 477, 12 pp., 1960.

The report is concerned with the vortex flow which arises when separation occurs at a highly swept leading edge. Low-speed measurements in air and water were made of the flow over flat plates at 15° incidence, each having a sharp leading edge of 65° sweep. The pressure and velocity distributions both along the axis of the vortex and for one cross section of the flow are presented together with a preliminary discussion of their significance.

From authors' summary by J. V. Becker, USA

1440. Kobayakawa, S., and Ting, L., Generalized integral relationship in flow fields with vortex sheets, *J. Aerospace Sci.* **27,** 11, 872–873 (Readers' Forum), Nov. 1960.

1441. Jones, E. E., The elliptic cylinder in a shear flow with hyperbolic velocity profile, *Quart. J. Mech. Appl. Math.* **12,** 2, 191–210, May 1959.

The fluid is assumed to be incompressible and inviscid and the flow is two-dimensional, steady and rotational. The stream func-

tion for the shear flow with hyperbolic velocity profile past an elliptic cylinder is determined in the form of an infinite series of Mathieu functions. It is found that the stagnation streamline of the flow is displaced toward a region of higher velocity, this displacement increasing (1) with the increase of the angle of incidence of the cylinder to the main stream, (2) as the stream becomes progressively nonuniform, and (3) with the increase of the minor axis length when the major axis length remains invariant. In each case, the displacement reaches a limiting value as the cylinder moves away from the axis of symmetry of the stream. The pressure coefficients and the resultant force end moment coefficients associated with the cylinder are also obtained, and are investigated numerically for the flat plate.

D. Gh. Ionescu, Roumania

1442. Szczeniowski, B., Solution of boundary problems in two-dimensional potential motion of incompressible perfect fluid (in English), *Arch. Budown Maszyn* **6**, 2, 207–257, 1959.

Author expresses stream function ψ in terms of arbitrary functions of $\alpha = x + iy$, $\beta = x - iy$. Condition that ψ should be real everywhere and constant on prescribed boundary limits form of arbitrary functions and then judicious choice leads to possible solutions. Various examples of flows past closed contours and through channels are given.

Reviewer notes that with this purely algebraic technique an *a priori* decision cannot be made as to whether other unwanted boundaries and singularities are present. However, method could be useful in simple cases. In sections 10, 11, author commits cardinal sin of prescribing both profile and one velocity component on it. This leads to a non-zero drag (not surprisingly) and a fruitless discussion of d'Alembert's paradox. This part is also subject of critical comment in attached note by editorial board.

H. C. Levey, Australia

1443. MacLellan, G. D. S., Mitchell, A. E., and Turnbull, D. E., Flow characteristics of piston-type control valves, Symposium on Recent Mechanical Engineering Developments in Automatic Control, Instn. Mech. Engrs., London, Jan. 5–7, 1960, 20 pp.

1444. Spence, D. A., A note on the recovery and Reynolds-analogy factors in laminar flat-plate flow, *J. Aerospace Sci.* **27,** 11, 878–879 (Readers' Forum), Nov. 1960.

1445. Zuber, N., On the variable-density single-fluid model for two-phase flow, *ASME Trans.* **β2C** (*J. Heat Transfer*), 3, 255–258 (Tech. Briefs), Aug. 1960.

Author discusses the similarity between Bankoff's model for turbulent co-current flow of liquid and gas and recent Russian papers on the correlation of mean vapor volume fraction and vapor volumetric flow concentration. Bankoff's model replaces the two-phase system with a single phase whose density is a function of radial position. The ratio of mean velocities of vapor and liquid is expressed as a function of the mean-vapor-volume fraction, α, by the relation, $u''/u' = \dfrac{1-\alpha}{K-\alpha}$ where K is a flow parameter dependent on pressure and mass flow rate. K ranges from 0.5 to 1.0 in circular pipes and averages around 0.89 for the steam-water system over a wide range of pressures and compositions.

Author shows that K may be identified with the ratio of mean-vapor-volume fraction and vapor-volumetric-flow concentration and is thus ultimately connected with the nonuniform distribution of the gas phase in the flow system. Recent Russian papers on steam-water and air-water mixtures indicate that K is almost independent of mass flow rate but is weakly dependent on pressure, ranging in value from 0.793 at 3 kg/cm² to 0.975 at 210 kg/cm² for the water steam system.

W. R. Foster, USA

1446. Mack, L. R., and Malsy, J., Experimental studies of an annular jet, State Univ. of Iowa, Iowa Inst. of Hydraul. Res. [Contract Nonr 1509(03)], 16 pp. + figs., Sept. 1960.

Lift-augmentation factors were computed from pressure measurements on the base plate of an annular-jet nozzle held stationary over land. A jet directed inwardly 30° from the vertical gave higher augmentations than 0° and 15° inclinations, no significant differences between the 0° and 15° inward discharge angles being noted. Over the range of exit gap widths tested, both the augmentation factor and the radial uniformity of the base-plate pressure distribution increased with increasing width of the annulus.

A rigidly supported 7-inch-diameter annular-jet nozzle discharging air vertically at three different rates through a 1/8-inch gap was towed at two altitudes and three speeds (including zero velocity) over initially quiescent water. For each of the 18 resultant runs the configuration of the water surface in the vicinity of the nozzle and the pressure distribution on the base of the nozzle were determined and plotted in the form of contour drawings. From the pressure distributions, the augmentation factor, the location of the center of pressure on the base plate, and the pitching moment on the base plate were computed for each run. In general, it was noted that all observed water-surface phenomena moved to the rear and became less pronounced in magnitude with increasing forward speed or with decreasing jet momentum flux. For all conditions of forward motion tested, pressures on the rear half of the base plate were larger than those on the front, thus giving rise to a "nose-down" pitching moment acting on the base of the nozzle. For the stationary runs the augmentation increased with decreasing jet momentum as predicted by Mack and Yen. For each combination of altitude and jet momentum flux, there appeared to be a forward speed for which the augmentation factor became a maximum, further increases in speed causing a reduction of augmentation from its peak value.

From authors' summary

1447. Bloom, M. H., On three-dimensional free-mixing, AFOSR TN 60-986 (Polyt. Inst. Brooklyn, Dept. Aero. Engng. Appl. Mech., PIBAL Rep. no. 474), 12 pp., Aug. 1960.

The quasi-two-dimensional nature of certain types of free three-dimensional viscous flows of a quasi-parallel nature is discussed. These flows are presumed to be describable in boundary-layer terms and are imbedded within flows which are essentially inviscid. In particular, flows of free-wake and jet type are considered. Flows of this character can originate, for example, from the boundary layers or shock-layers of non-axially-symmetric bodies or from non-axially-symmetric jet-producers.

From author's summary

Compressible Flow (Continuum and Noncontinuum Flow)

(See also Revs. 1436, 1478, 1481, 1482, 1483, 1485, 1486, 1487, 1493, 1494, 1529, 1530, 1556, 1565, 1568, 1590, 1603, 1620, 1622, 1626, 1636, 1643, 1644, 1655, 1659)

1448. Howes, W. L., Distribution of time-averaged pressure fluctuations along the boundary of a round subsonic jet, NASA TN D-468, 20 pp., Oct. 1960.

A semiempirical analysis of the equation for incompressible fluctuations in a turbulent fluid, using similarity relations for round subsonic jets with uniform exit velocity, is used to predict the shape of the time-averaged fluctuation-pressure distribution along the mean-velocity boundary of jets. The predicted distribution is independent of distance downstream of the nozzle exit along the mixing region, inversely proportional to the distance down-

stream along the region of mean-velocity self-preservation, and proportional to the inverse square of the distance downstream along the fully developed region.

Experimental results were in fair agreement with the theory. However, the measured fluctuation-pressure distributions were found to be very sensitive to changes in jet temperature and jet-nozzle profile, especially near the nozzle. These factors are not included in the theory. Increased jet temperatures produce increased pressure fluctuations and violation of similarity conditions. Nozzle-profile modifications may lead to violation of the uniform-exit-velocity requirement imposed in the theory.

From author's summary

1449. Smith, W. G., and Lazzeroni, F. A., Experimental and theoretical study of a rectangular wing in a vortical wake at low speed, NASA TN D-339, 33 pp., Oct. 1960.

A systematic study has been made, experimentally and theoretically, of the effects of a vortical wake on the aerodynamic characteristics of a rectangular wing at subsonic speed. The vortex generator and wing were mounted on a reflection plane to avoid body-wing interference. Vortex position, relative to the wing, was varied both in the spanwise direction and normal to the wing. Angle of attack of the wing was varied from $-4°$ to $+6°$. Both chordwise and spanwise pressure distributions were obtained with the wing in uniform and vortical flow fields. Stream surveys were made to determine the flow characteristics in the vortical wake.

The vortex-induced lift was calculated by several theoretical methods including strip theory, reverse-flow theory, and reverse-flow theory including a finite vortex core. In addition, the Prandtl lifting-line theory and the Weissinger theory were used to calculate the spanwise distribution of vortex-induced loads. With reverse-flow theory, predictions of the interference lift were generally good, and with Weissinger's theory the agreement between the theoretical spanwise variation of induced load and the experimental variation was good. Results of the stream survey show that the vortex generated by a lifting surface of rectangular planform tends to trail back streamwise from the tip and does not approach the theoretical location, or centroid of circulation, given by theory. This discrepancy introduced errors in the prediction of vortex interference, especially when the vortex core passed immediately outboard of the wing tip.

The wake produced by the vortex generator in these tests was not fully rolled up into a circular vortex, and so lacked symmetry in the vertical direction of the transverse plane. It was found that the direction of circulation affected the induced loads on the wing either when the wing was at angle of attack or when the vortex was some distance away from the plane of the wing.

From authors' summary

1450. Grant, F. C., and Sevier, J. R., Jr., Transonic and supersonic wind-tunnel tests of wing-body combinations designed for high efficiency at a Mach number of 1.41, NASA TN D-435, 82 pp., Oct. 1960.

Wind-tunnel force tests of a number of wing-body combinations designed for high lift-drag ratio at a Mach number of 1.41 are reported. Five wings and six bodies were used in making up the various wing-body combinations investigated. All the wings had the same highly swept discontinuously tapered planform with NACA 65A-series airfoil sections 4% thick at the root tapering linearly to 3% thick at the tip. The bodies were based on the area distribution of a Sears-Haack body of revolution for minimum drag with a given length and volume. These wings and bodies were used to determine the effects of wing twist, wing twist and camber, wing leading-edge droop, a change from circular to elliptical body cross-sectional shape, and body indentation by the area-rule and streamline methods. The supersonic test Mach numbers were 1.41 and 2.01. The transonic test Mach number range was from 0.6 to 1.2.

For the transition-fixed condition and at a Reynolds number of 2.7×10^6 based on the mean aerodynamic chord, the maximum value of lift-drag ratio at a Mach number of 1.41 was 9.6 for a combination with a twisted wing and an indented body of elliptical cross section.

The tests indicated that the transonic rise in minimum drag was low and did not change appreciably up to the highest test Mach number of 2.01. The lower values of lift-drag ratio obtained at a Mach number of 2.01 can be attributed to the increase of drag due to lift with Mach number.

From authors' summary

1451. Ryzhov, O. S., and Shefter, G. M., The approximate structure of one class of nonsteady transonic flow, *Soviet Phys.-Doklady* **5**, 1, 1-4, July/Aug. 1960. (Translation of *Dokladi Akad. Nauk SSSR (N.S.)* **130**, 2, 276-279, Jan./Feb. 1960 by Amer. Inst. Phys., Inc., New York, N.Y.)

Authors obtain an accurate particular solution for an equation of flow which is satisfied by the velocity potential of non-steady transonic flow when the flow parameters vary sufficiently rapidly with time. Solution obtained describes potential flow without pressure discontinuities and which contains the variations with time of local supersonic zones. In this solution, the form of the nozzle is not prescribed in advance, but must be chosen relative to the solution obtained, and it may vary with time in the general case.

From authors' summary

1452. Softley, E. J., On transonic flow about two parallel bodies of revolution, AFOSR TN 60-706 (Brown Univ., Div. Engng. TR-W7-33), 57 pp., May 1960.

Potential theory, based on linearization of the transonic flow equation, is applied to the case of two slender bodies of revolution, situated with parallel axes and within a region where interference effects cannot be neglected. The forces on the bodies are discussed and, as an example, the drag of one body is computed and compared with experimental results obtained by G. Drougge.

From author's summary

1453. Drougge, G., and Landahl, M. T., An experimental investigation of supersonic wing-body interference and comparison with second-order theory, AFOSR TN 60-257 (Flygtek. Forsoksanst. Rep. AU-11-93:2), 25 pp., Dec. 1959.

Paper describes tests on four model wing-body combinations at Mach numbers of 3 and 4. Test sections of two small wind tunnels were calibrated, cone cylinder models inserted and nonuniform flow field behind shock waves explored. Basic pressure distributions on four wings (no body interference) were obtained, and finally pressure distributions measured on wings located at different positions in cone-cylinder field.

Wing-alone results compare well with theoretical prediction when wing thickness is increased by boundary-layer displacement thickness. Body-alone results compare well with method of characteristics calculations and supersonic similarity law. Simple formula for second-order interference pressure coefficient, derived by Landahl, Drougge and Bean, gives fair prediction of $M = 3$ wing-body pressures, good prediction at $M = 4$.

J. H. Horlock, England

1454. Seegmuller, G., Numerical investigations on the solution of the non-linear equation of supersonic flow by finite difference methods, Part 1, AFOSR TN 60-255 (Mathematical Institute, Technische Hochschule Munchen TN 10), 45 pp., Nov. 1959.

A finite difference approximation to the method of characteristics is used to obtain solutions to plane and axisymmetric steady irrotational isentropic flows of ideal gases. Five iterative pro-

cedures are described for the solution of the resulting difference equations and each is exemplified by numerical results for a single point. Comparison is made between the procedures and particular attention is paid to convergence, effect of rounding errors, truncation errors, and application in the neighborhood of an axis of symmetry.

It is shown how the true solution may be extrapolated from results for different mesh sizes and how the accuracy of the extrapolation may be assessed.

Finally, the five algorithms used for the iterations are programmed in ALGOL notation. S. Kirkby, England

1455. Sugo, M., The second-order theory of the supersonic three-dimensional thin-wing-expansion, *Trans. Japan Soc. Aero. Space Sci.* **2,** 2, 1–10, 1959.

Investigations are made to generalize the second-order theory in the inviscid, compressible, and irrotationally three-dimensional field. The method used is an iteration process which converges at least on the compression side of the wing. Since the applicable limit of the given theory is shown as the condition used to determine the particular solution the method is invalid for the expansion side of the wing.

To avoid higher-order singularities in his adopted iteration process author introduces the P(oincaré)-L(ighthill)-K(uo) method. This analytical technique circumvents the difficulty of ascending order of singularities in the second-order term. Analogous to van Dyke's consideration the particular solution of the second-order equation is given in the form of a combination of linear potentials. For cases of the subsonic and supersonic leading edges, the pressure distributions are calculated and the lift curve slope is also given for the latter case. Discussion of the transonic leading-edge airfoil by this theory is difficult.

The results are compared, for example, with those of Moore and Fowell and, especially for the lift curve slope, with those of an experiment by Lammpert. G. A. Altmann, Germany

1456. Grozdkov, N. N., Calculation of the friction of a wedge being flown around by a turbulent gas flow (in Russian), *Trudi Voronezhsk. In-ta* **42,** 2, 3–5, 1956; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5169.

The paper gives the results of the calculations for the turbulent friction of a wedge, flown about by a supersonic flow. In making the calculations use was made of the formula derived by the author jointly with M. F. Shirokov [*Vestn. Mosk. Un-ta*, no. 10, 105–115, 1953]. It was demonstrated that the results of the calculations by the given formula agreed satisfactorily with the results of calculations made in consonance with other theories.

I. N. Sokolova
Courtesy Referativnyi Zhurnal, USSR

1457. Osborne, R. S., and Kelly, T. C., A note on the drag due to lift of delta wings at Mach numbers up to 2.0, NASA TN D-545, 14 pp., Nov. 1960.

In order to indicate the effects of Reynolds number and other variables on the drag due to lift of delta wings for Mach numbers up to 2.0, the results of several investigations of wing-body combinations having plane delta wings with aspect ratios from 2 to 4 have been assembled for comparison and brief analysis.

The effects of Reynolds number, leading-edge radius, and thickness ratio could generally be correlated with Reynolds number based on the leading-edge radius as a parameter. The effects of leading-edge Reynolds number on drag due to lift were large at Mach numbers less than 0.25. However, with increases in Mach number, the effects decreased and were almost negligible at a Mach number of 2.0. The effects of aspect ratio and trimming were large, as would be expected.

It was indicated, at least for subsonic and transonic speeds, that improvement in the drag due to lift might be obtained from wing modifications designed to inhibit flow separation.

From authors' summary

1458. Romig, Mary F., Conical flow parameters for air in dissociation equilibrium, Convair Res. Rep. 7, 64 pp., May 1960.

The Taylor-Maccoll equation describing supersonic flow about cones has been integrated numerically for cone semi-vertex angles from 20° to 50°, velocities from 10,000 to 32,000 ft/sec, free stream pressures from 10^{-4} to 10^{-1} atmospheres, and a nominal free-stream temperature of 491.7°R (273.16°K). The air between the shock wave and the body is assumed to be in instantaneous thermal and chemical equilibrium.

Values are tabulated for the aerodynamic and thermodynamic flow properties, electron density, velocity, and Mach number directly behind the shock wave and on the cone surface streamline.

Use of the hypersonic similarity parameter, $M_1 \sin \theta_b$, permits correlation of these properties with an error of less than 1% in all cases. Graphs are presented of the conical flow properties as functions of $M_1 \sin \theta_b$ and free-stream pressure. Empirical equations based on the similarity rule are given for the shock-wave angle and pressure coefficient. The effect of dissociation on shock detachment is discussed briefly.

From author's summary

1459. Robins, A. W., Harris, R. V., Jr., and Jackson, C. M., Jr., Characteristics at Mach number of 2.03 of a series of wings having various spanwise distributions of thickness ratio and chord, NASA TN D-631, 26 pp., Oct. 1960.

A series of semispan wing models having various spanwise distributions of both thickness ratio and chord but having the same effective thickness ratio was tested in the Langley 4- by 4-foot supersonic pressure tunnel at Mach number 2.03 and Reynolds numbers from 1.9×10^6 to 6.5×10^6 based on mean geometric chord. The data indicate that the complex wing forms with thickened roots, extended root chords, and higher volumes show appreciably lower zero-lift wave drag coefficients than the plain swept wings. A calculative technique for the determination of wave drag has been applied to one of the complex wings of the series and good agreement is shown with experimental results. The complex wing forms showed higher drags due to lift than the plain swept wings.

From authors' summary

1460. Ol'khovskii, I. I., General hydrodynamic theory of a supersonic interferometer, *Soviet Phys.-Doklady* **5,** 1, 105–109, July/Aug. 1960. (Translation of *Doklady Akad. Nauk SSSR (N.S.)* **130,** 4, 747–750, Jan./Feb. 1960 by Amer. Inst. Phys., Inc., New York, N.Y.)

In previous papers, author proposed a method of solution of the general equations of hydrodynamics, and considered the question of boundary conditions of the Maxwell type. He also solved the problem of a supersonic interferometer for the 4-moment approximation, i.e., without taking heat conduction into account. In the present work, he gives a general hydrodynamic theory for a supersonic interferometer for a 5-moment approximation, i.e., for an approximation that takes the heat conduction of the gas and the boundary into account.

From author's summary

1461. Kogan, A., and Betser, A. A., On rotational supersonic flow past thick airfoils—tables, AFOSR TR (Final) 60-135 (Techn. Res. Develop. Foundation, Haifa, Israel), 75 pp., June 1960.

Certain shock-wave parameters, which appear in the calculation of pressure distribution on airfoils in a supersonic rotational flow, are presented in tabular form. The method underlying the use of

the tables is summarized and discussed. A numerical example illustrates the procedure to be followed in the application of the tables. From authors' summary

1462. Chow, W. L., Korst, H. H., and Tsung, C. C., Truncated cone in supersonic flight at zero angle of attack—Surface pressure coefficients, drag coefficients, and shock front configurations, Univ. Illinois, Mech. Engng. Dept. TN 392-6, 4 pp. + charts, Jan. 1960.

1463. Hartsell, C. W., Derivation of an analytical expression for supersonic spillage of single-cone inlets, *J. Aerospace Sci.* **27**, 10, 788–789 (Readers' Forum), Oct. 1960.

1464. Trimpi, R. L., and Jones, R. A., A method of solution with tabulated results for the attached oblique shock-wave system for surfaces at various angles of attack, sweep, and dihedral in an equilibrium real gas including the atmosphere, NASA TR R-63, 157 pp., 1960.

A new method is derived for solving the attached oblique shock-wave system for surfaces at various angles of attack, sweep, and dihedral in any real gas in equilibrium. Results are tabulated for the following ranges: angles of attack, 0 to 65°; angle of sweep, 0 to 75°; angle of dihedral, 0 to 30°; Mach number, 3 to 30; and "effective specific heat ratio" parameter, 1.10 to 1.67. Both the method and the tabulated solutions are easily adaptable to flight in any gas or in the atmosphere of any planet. An illustrative example is presented based on the ARDC 1956 model atmosphere.

From authors' summary by A. J. Chapman, USA

1465. Sammonds, R. I., The shock-wave patterns on a cranked-wing configuration, NASA TN D-346, 12 pp., Nov. 1960.

The shock-wave patterns of a complex configuration with cranked cruciform wings and a cone-cylinder body were examined to determine the interaction of the body bow wave with the flow field about the wing. Also of interest was the interaction of the forward (76° sweptback) wing leading-edge wave with the rear (60° sweptback) wing leading-edge wave. The shadowgraph pictures of the model in free flight at a Mach number of 4.9, although not definitive, appear to indicate that the body bow wave crosses the outer wing panel after first being refracted either by the leading-edge wave of the 60° sweptback wing or by pressure fields in the flow crossing the wing. From author's summary

1466. Manheimer-Timnat, Y., Ionization measurements in shocked gases, *Bull. Res. Council Israel* **8**C, 1, 53–55, Feb. 1960.

1467. Henderson, A., Jr., Investigation of the flow over simple bodies at Mach numbers of the order of 20, NASA TN D-449, 17 pp., Aug. 1960.

The paper gives a brief review of experimental and theoretical pressure distributions on simple bodies in air and helium at hypersonic speeds, at high Reynolds numbers. It is shown that (1) pressures on blunt noses of widely varying shapes are well predicted ($M = 19$) by matched Newtonian-Prandtl-Meyer distributions; (2) induced pressures at $x/d \geq 1$ from nose of a hemisphere cylinder are little affected by Re_d at $Re_d \geq 100{,}000$, $M > 15$; (3) pressures downstream of the shoulder are independent ($M = 21$) of the nose shape; (4) blast wave parameter $(x/d)/M^2\sqrt{C_D}$ correlated well the pressures on cylinders at $x/d > 3$, $M = 21$, $\gamma = 5/3$, for a wide range of nose drag, $0.2 \leq C_D \leq 1.2$; (5) for two-dimensional bodies, the induced pressure effects on C_N, C_m and cp are large at low incidence (5 deg) and high M (=20).

The concluding statement, "The designer has adequate means at his disposal for calculating the inviscid direct and induced pressures on simple axisymmetric bodies at zero angle of attack," needs, in the reviewer's opinion, some qualification since the usefulness of available theories and correlations depends on the body geometry, Mach number and x/d range considered.

J. Lukasiewicz, USA

1468. Patterson, G. N., Transport properties of free molecule (Knudsen) flow, Univ. Toronto, Inst. Aerophys. Rev. no. 11, 31 pp., Mar. 1958.

The paper is a review of free molecule flow theory and experiment: from a list of 22 references the latest are dated 1957. Regimes of low density flow are defined and the current state of knowledge about gas/surface interactions discussed. Molecular velocity distribution function is defined as density of points in velocity space rather than the more usual density in phase space. Useful sections describe theory of free molecule probes (pressure and equilibrium temperature types) together with their use in shock transition investigations. Text is marred by misprints which would make understanding difficult for newcomers to the subject. Statement halfway down p. 17 implies that shock thickness is 5000 times mean free path instead of 5000 times probe diameter.

J. F. Clarke, England

1469. Sunavala, P. D., Dynamics of the turbulent free jet, *J. Sci. Indust. Res., India* **19 B** (*Physical Sciences*), 2, 35–45, Feb. 1960.

Author reports on thrust measurements of small converging nozzles (1/16-inch to 3/8-inch diameter). A one-dimensional analysis is used to correlate axial temperature data with thrust data for subsonic nozzle discharges.

Reviewer believes that title may be misleading since it is so general that readers may feel entitled to a report on more detailed analyses and experiments than those discussed in the paper.

R. H. Page, USA

1470. Troshin, V. L., Flow of a gas jet out of a channel past a flat plate, *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **23**, 4, 1090–1096, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Falkovich [*Prikl. Mat. Mekh.* **21**, 4, 459–464, 1957] introduced a method which extends Chaplygin's hodograph method to jet problems having more than one characteristic speed. This is applied in the present paper to the case of a gas jet flowing at subsonic speed out of a two-dimensional channel past a flat plate normal to the jet and at a distance s from the channel mouth. The two characteristic speeds are v_1 far upstream in the channel and v_2 on the free surfaces of the jet. By varying s and l/d, where l is the plate width and d the channel width, several problems are solved: (1) the situation described above, in which the jet is divided by the plate into two jets which, far downstream, have the inclinations $+m$; (2) $v_1 = v_2$ gives a free jet striking a plate (Chaplygin's problem); (3) For $m = (1/2)$ the plate becomes an infinite wall in front of the channel; (4) The plate inside the channel at a finite distance from the mouth. The last case is also the solution to flow out of a vessel of finite width and infinite length.

A. Roshko, USA

1471. Portnov, I. G., Calculations for steady regimes of the work of supersonic gas ejectors (in Russian), Means for developing the gas industry (Puti Razvitiya Gaz. Prom-sti SSSR), Moscow, Gostoptekhizdat, 1958, 81–85; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 4911.

Equations for ejection are put forward applicable to an ejector with a cylindrical mixing chamber. In the derivation of the equations use is made of the hypothesis on the existence of sections of equal pressure, up to which point the low pressure and high pressure streams of gas move separately, without mixing. A comparison is made between the calculation data and the results of industrial tests of a gas ejector at the Elshan-Kurdyumsk gas works

in Saratovsk province. The author states that in steady regimes of work of the gas ejector the divergence of the calculation data and the experimental is not great. The observed difference can be explained, in the author's view, as due to the unsteady work of the gas ejector.

Yu. A. Lashkov

Courtesy Referativnyi Zhurnal, USSR

1472. Ryzhov, O. S., and Shefter, G. M., On unsteady gas flows in Laval nozzles, *Soviet Phys.-Doklady* **4**, 5, 939–942, Mar./Apr. 1960. (Translation of *Doklady Akad. Nauk SSSR (N.S.)* **128**, 3, 485–487, Sept./Oct. 1959 by Amer. Inst. Phys., Inc., New York, N.Y.)

An exact solution, containing four arbitrary functions of time, of the nonlinear equation of unsteady potential, transonic, small perturbed flow is obtained. It is physically interpreted in relation with unsteady flow in Laval nozzles in the vicinity of sonic surface.

J. P. Guiraud, France

1473. Sidorov, A. F., Nonstationary potential flow of a polytropic gas with degenerate hodograph, *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **23**, 5, 1334–1339, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N.Y.)

Author proposes a method of investigating nonstationary potential flows of a polytropic gas. For the general three-dimensional problem, an auxiliary function is introduced, and the hodograph transformation leads to a nonlinear second-order partial differential equation for the sound speed. If one adds the restriction that a functional relationship $u_3 = \psi(u_1, u_2)$ exists among the velocity components, a similar procedure leads to two types of solutions. One is the two-dimensional result derived elsewhere. In the other case, the function ψ must satisfy a partial differential equation which describes developed surfaces in the hodograph. After choosing a solution ψ, one must verify the compatibility, and then find the solution, of a pair of partial differential equations for the auxiliary function. In principle this procedure allows one to determine the flow field which corresponds to a given function ψ. The author points out that a linear function ψ satisfies all the requirements, but no further discussion of example, or of methods for solving the equations, is given.

A. F. Messiter, USA

1474. Brun, E. A., Aero-thermo seminar, arranged by the Faculté des Sciences de Paris, 1957–1958 (in French), Publ. Scient. Tech. Min. Air, France NT 85, 170 pp., 1959.

Nine lectures sponsored by Science Faculty, Univ. of Paris, Dec. 1957–May 1958. First four are related to rarefied gasdynamics presented by L. Talbot: (1) "Recent research in rarefied gasdynamics"; (2) "Shock wave structure"; (3) "Couette flow in rarefied gas"; (4) "Pressures on cones and flat plates in hypersonic flow." Coverage of these topics may be found in "Rarefied gas dynamics" (in English), Pergammon, 1960, edited by F. M. Devienne, which contains papers of First International Symposium on Rarefied Gasdynamics at Nice. (5) E. A. Eichelbrenner, "Three-dimensional laminar boundary layer," shows how certain three-dimensional fields may be studied using an essentially two-dimensional analysis for various axisymmetric bodies at incidence. This analysis relies on various techniques developed for boundary-layer flow over yawed bodies and indicates how and where they are applicable. Specific examples for ellipsoids of revolution and flattened ellipsoids at incidence are carried out to establish the behavior of the flow in regions where laminar separation occurs. Using various flow visualization techniques in the hydrodynamic tunnel at Châtillon the surface flows indicate a qualitative check of the method. Included is a discussion of methods for predicting transition for various bodies and some experimental results presented for ellipsoids. (6) C. Bory, "Individual characteristics of matter in flows," presents a theoretical study which deals with the dissociation of fluid particles and their effect on the motion of the fluid. (7) A. K. Oppenheim, "On the development of detonations in a gas," analyzes the process for the formation of the wave. He presents an analytical method which allows for the evaluation of effects due to the propagation of waves and the interaction phenomena observed in a duct of constant cross section for the transitional regime as the deflagration approaches a detonation. (8) M. Cloupeau, "Shock waves produced in gas-filled tubes by condenser discharges," examines experimentally continuum optical radiation emitted when a capacitor is discharged across a gas filled tube. In particular, radiation reflected back into the plasma from the walls of the tube has been observed. (9) R. DerAgobian, "R.F. study of shock waves produced by condenser discharge in a gas." Preliminary experiments show a correlation between optical and microwave (10 KMc) phenomena in a gas discharge.

S. Lampert, USA

1475. Ridyard, H. W., A comparison of blunt-body flow-field results, *J. Aerospace Sci.* **27**, 10, 789–791 (Readers' Forum), Oct. 1960.

Boundary Layer

(See also Revs. 1444, 1474, 1548, 1569, 1575, 1590, 1600, 1603, 1619, 1621, 1625, 1647)

1476. Wilkinson, J., The steady incompressible boundary-layer flow past a flat plate with a parabolic leading edge, *Quart. J. Mech. Appl. Math.* **13**, 2, 199–209, May 1960.

Paper obtains an approximate solution of the incompressible Navier-Stokes equations for the flow past a flat plate with a parabolic leading edge using paraboloidal-type coordinates. Method is a three-dimensional extension of the work of Kaplun [*ZAMP* **5**, 1954; AMR **8** (1955), Rev. 458] on optimal coordinate systems. The improved boundary-layer-type solution obtained is compared with Moore's simpler method [NACA TN no. 2279] with the conclusion that the latter is a satisfactory solution of the boundary-layer equations for a flat plate with arbitrary but continuous leading-edge geometry. Assessment of the accuracy of author's approximation is made by comparison of magnitudes of the neglected terms in the Navier-Stokes equations, and reviewer questions the validity of the conclusion derived in this way that on the surface the approximation holds up to and including the leading edge.

J. A. Laurmann, USA

1477. Sinbel, M. A. Boundary-layer solution for a flow in a diverging passage having a swirl component, *ASME Trans.* **81** E (*J. Appl. Mech.*), 4, 477–484, Dec. 1959.

Author attacks the problem of solving equations of the boundary-layer flow in a diverging passage having a swirl component. The main goal is to estimate the boundary-layer thickness owing to translational and rotational motion. The main stream possesses both swirl and axial velocity. The development of the boundary layer in a conical duct is influenced by centrifugal action in addition to the pressure and viscous forces. Equations of flow of viscous, incompressible fluid in spherical polar coordinates form the starting point; by the standard technique the differential equations are transformed into integral forms. The Prandtl form of the turbulent stress tensor is assumed and the two velocity components (radial and perpendicular to axial planes) are expressed in terms of polynomials, each having four terms (an analogy to Pohlhausen's technique). A sufficient number of boundary conditions assures the unique determination of the 8 coefficients appearing in the polynomials. Next, author introduces nondimensional variables, referred to two characteristic quantities: nondimensional boundary-

layer thickness and the deviation of direction of the surface drag from that of the pressure gradient. A comparison of the present resulting equations with those given previously by A. M. Binnie and D. P. Harris ["The application of boundary-layer theory to swirling liquid flow through a nozzle," *Quart. J. Mech. Appl. Math.* **3**, pt. 1, p. 89, 1950] and by G. I. Taylor ["The boundary layer in the converging nozzle of a swirl atomiser," Ibid, part 2, p. 129, 1950] shows a good agreement. The semi-empirical solution of the system of equations was compared with the experimental data, in which the swirl was introduced into flows inside cones by a spiral case, the side wall of which was a logarithmic spiral. The agreement is fair.

M. Z. v.Krzywoblocki, USA

1478. Hill, F. K., Turbulent boundary layer measurements at Mach numbers from 8 to 10, *Physics of Fluids* **2**, 6, 668–680, Nov./Dec. 1959.

Detailed experimental studies were made on the velocity and temperature profiles in the boundary layer on the axisymmetric nozzle wall of hypersonic nitrogen gas flow at Mach numbers from 8 to 10. At these high Mach numbers the laminar sublayer occupies a considerable portion of the boundary layer up to about one-third the total thickness, and hence its effect is significant. Distinct transition from the laminar sublayer to turbulent portion was observed. The velocity distribution in the turbulent portion was fitted well by the one-fifth power formula. Negative or positive pressure gradient provided only a limited influence. In correlating the skin-friction coefficient, the momentum-thickness Reynolds number was not a particularly good parameter. Heat-transfer coefficients were about 20% less than theoretical predictions. A physically more realistic theory in dealing with hypersonic turbulent boundary layers seems to be needed.

F. R. Hama, USA

1479. Solodkin, E. E., and Ginevskii, A. S., Determination of the characteristics of a turbulent boundary layer and the resistance of long axisymmetrical bodies (in Russian), *Trudi Naucbno-Tekbn. O-va Sudostroit. Promsti* **7**, 2, 81–106, 1957; *Ref. Zb. Mekb.* no. 5, 1959, Rev. 5173.

The solution of the problem regarding the turbulent boundary layer on a standard smooth plate is generalized to cover the case of the longitudinal flowing about of a cylinder. The authors in carrying out the solution do not make use of the assumption of the smallness of the thickness of the boundary layer δ in comparison with the cylinder's radius r. The "universal" constants R and α are taken to be equal to their values for the plate because of lack of experimental data. The calculations given show that the influence of the transverse curvature produces an increase in frictional resistance. These results are in agreement with known experiments. The obtained solution is utilized for the derivation of a system of calculation for a turbulent boundary layer on a long body of rotation at small longitudinal pressure gradients, on the assumption that the small longitudinal pressure gradients exhibit no influence on the form of the profile of velocities in the boundary layer. Curves are furnished to shorten the calculations. An expression is found, by the use of the method of impulses, for the coefficient of resistance for the profile of the body of rotation, which with $r \to \infty$ passes over to the known Squire and Young relation.

G. N. Tkachuk

Courtesy Referativnyi Zhurnal, USSR

1480. Landweber, L., Reanalysis of boundary-layer data on a flat plate, State Univ. of Iowa, Iowa Inst. of Hydraul. Res. [Contract Nonr 1611(01)], 10 pp. + figs., Oct. 1960.

Procedures for correcting velocity-profile data, obtained with round pitot tubes, for pitot-tube displacement effect and turbulence fluctuations are described and applied to several sets of boundary-layer data.

Direct measurements of shear stress on a flat plate in zero pressure gradient by different investigators and values computed using the complete momentum equation (including the effects of turbulence and residual pressure gradients) are in excellent agreement.

The law of the wall appears to be a universal similarity relation for values of y^* less than 100.

A procedure is given for deriving the calibration curve for Preston tubes from the law of the wall and applied to a mean curve for the law for values of y^* from 10 to 100. This calibration curve lies between those of Preston and Relf and is closer to the former. This last result is discussed and justified.

From author's summary

1481. Pallone, A., Non-similar solutions of the compressible laminar boundary layer equations, with applications to the upstream transpiration cooling problem, AVCO Corp., Research & Advanced Develop. Div., Wilmington, Mass., RAD-TR-9-60-8, 35 pp., May 1960.

In this paper a new method of obtaining nonsimilar solutions of the boundary-layer equation with and without a streamwise pressure gradient is developed and applied to predict the boundary-layer characteristics downstream of the porous region of an injection-cooled surface. The method combines the Dorodnitzin integration scheme with the Pohlhausen approach and is formulated along the following lines. The viscous domain is divided into N curvilinear strips. The governing equations are then integrated along the coordinate normal to the body from the surface to the boundary of each strip. As a result, one obtains a set of independent integro-differential relations. The integration is carried out by expressing the integrands as polynomials, the coefficients of which are functions of the unknown values of the velocity and temperature on the strip boundaries as well as of the imposed boundary condition at the wall and at the outer edge. The set of ordinary first-order differential equations obtained after the integration may be solved for given initial conditions by a numerical integration scheme such as the Runge-Kutta method. Several numerical examples for an ideal gas are presented.

From author's summary by G. Sestini, Italy

1482. Poots, G., A solution of the compressible laminar boundary layer equations with heat transfer and adverse pressure gradient, *Quart. J. Mecb. Appl. Math.* **13**, 1, 57–84, Feb. 1960.

Stewartson transformation [AMR **3**(1950), Rev. 2019] reduces equations to two coupled partial differential equations of semi-incompressible form. These are solved for small values of longitudinal coordinate X by means of series expansions, and solution is continued numerically by Hartree-Womersley method.

Approximate solution is also developed by Pohlhausen technique. This is development of method used by Tani for purely incompressible flow, and involves momentum integral equation, kinetic-energy integral equation and thermal-energy integral equation. Results are satisfactorily compared with those given by numerical solutions above.

N. Curle, England

1483. Nagamatsu, H. T., and Li, T. Y., Hypersonic flow near the leading edge of a flat plate, *Physics of Fluids* **3**, 1, 140–141 (Letters to the Editor), Jan./Feb. 1960.

Some brief preliminary results are presented on the possibility of the existence of a very interesting slip-flow phenomenon at the leading edge of a flat plate at hypersonic Mach numbers in a continuum flow. The results are based on schlieren photographs (a flow at $M = 19.3$ is shown) of the shock boundary-layer interaction region on a flat plate obtained in a hypersonic shock tunnel. (The details of the measurements of the flow quantities used in computing the Mach and Reynolds numbers are not given.) The authors point out that a great deal of experimental and analytical work is still required to establish the properties of this type of flow.

I. I. Glass, Canada

1484. Freeman, N. C., and Lam, S. H., On the Mach number independence principle for a hypersonic boundary layer, AFOSR TN 59-728 (Princeton Univ., Dept. Aero. Engng. Rep. no. 471), 15 pp., July 1959.

The Mach number independence principle for a hypersonic viscous boundary layer postulated by Hayes and Probstein [AMR 13(1960), Rev. 2988] is considered for the case of a flat plate with no heat transfer. The structure of the boundary layer is given and the transition solution in the neighborhood of the edge of the boundary layer is constructed. This transition layer, in which the boundary layer adjusts to the free-stream conditions, is shown to have a thickness scale proportional to $(M_\infty)^{-1-S}$ as originally suggested (S is the exponent of viscosity law). The values of density, temperature and velocity across this layer are determined explicitly in terms of velocity and implicitly as functions of boundary-layer coordinates.

From authors' summary by L. N. Tao, USA

1485. Lunkin, Yu. N., Boundary-layer equations and their boundary conditions in the case of motion at supersonic velocities in a moderately rarefied gas, NASA TT F-28, 12 pp., May 1960.

....Following Prandtl's boundary-layer theory, equations are derived for a moderately rarefied gas which differ from the ordinary Prandtl equations by the presence of supplementary terms that contain higher derivatives of velocity and temperature; in the equations derived the normal pressure gradient differs from zero and is expressed by supplementary terms. The boundary conditions for these equations are found with the aid of kinetic theory; they are a generalization of Maxwell's... and Smoluchowski's... conditions for motion at a supersonic velocity. The velocity and height limits of applicability of the equations derived are set forth.

From author's summary by T. C. Adamson, Jr., USA

1486. Scala, S. M., Hypersonic viscous shock layer, *ARS J.* **29**, 7, 520-522 (Tech. Notes), July 1959.

Author treats problem of defining transition to continuum flow pertinent to the case of hypersonic lifting vehicles. Such vehicles composed of axially symmetric and two-dimensional elements are considered. The analysis is restricted to flight Mach numbers between 16 and 24 and to wall temperatures between 1000 and 3000 degrees R. Based on a merged layer parameter at the hypersonic stagnation point, limits of applicability of the standard aerodynamic technique of separating the flow field are given for cases of high altitude hypersonic flight. It is shown that a small nose radius promotes the formation of a viscous shock layer. The trends given by the analysis indicate two-dimensional lifting surfaces can be thinner than the center body and still operate in the same flow regime. R. W. Detra, USA

1487. Dorand, J.-F., Investigation of the boundary layer along a flat plate at supersonic flow, including the effects of artificially created transition (in French), Publ. Scient. Tech. Min. Air, France no. 355, 106 pp., 1959.

In a fine study author presents the results of explorations in the dynamic and thermal boundary layer along a horizontal flat plate at 1.85 Mach and $0.8-2.0 \times 10^6$ Reynolds. The most interesting observation made was the occurrence of laminar boundary-layer separation of the closed type, preceding laminar-turbulent transition. Author proves conclusively that this phenomenon cannot be attributed to experimental error or misinterpretation.

The study includes measurements behind transition-promoting wires glued to the plate surface. J. K. Yap, Holland

1488. Boltz, F. W., Kenyon, G. C., and Allen, C. Q., Effects of sweep angle on the boundary-layer stability characteristics of an untapered wing at low speeds, NASA TN D-338, 77 pp., Oct. 1960.

An investigation was conducted in the Ames 12-foot low-turbulence pressure tunnel to determine the effects of sweep on the boundary-layer stability characteristics of an untapered variable-sweep wing having an NACA 64_1A015 section normal to the leading edge. Pressure distribution and transition were measured on the wing at low speeds at sweep angles of 0°, 10°, 20°, 30°, 40°, and 50° and at angles of attack from −3° to 3°. The investigation also included flow-visualization studies on the surface at sweep angles from 0° to 50° and total pressure surveys in the boundary layer at a sweep angle of 30° for angles of attack from −12° to 0°.

It was found that sweep caused premature transition on the wing under certain conditions. This effect resulted from the formation of vortices in the boundary layer when a critical combination of sweep angle, pressure gradient, and stream Reynolds number was attained. A useful parameter in indicating the combined effect of these flow variables on vortex formation and on beginning transition is the crossflow Reynolds number. The critical values of crossflow Reynolds number for vortex formation found in this investigation range from about 135 to 190 and are in good agreement with those reported in previous investigations. The values of crossflow Reynolds number for beginning transitions were found to be between 190 and 260. For each condition (i.e., development of vortices and initiation of transition at a given location) the lower values in the specified ranges were obtained with a light coating of flow-visualization material on the surface.

A method is presented for the rapid computation of crossflow Reynolds number on any swept surface for which the pressure distribution is known. From calculations based on this method, it was found that the maximum values of crossflow Reynolds number are attained under conditions of a strong pressure gradient and at a sweep angle of about 50°. Due to the primary dependence on pressure gradient, effects of sweep in causing premature transition are generally first encountered on the lower surfaces of wings operating at positive angles of attack.

From authors' summary

1489. Potter, J. L., and Whitfield, J. D., Relation between transition location and pressure in separated shear flows, *J. Aerospace Sci.* **27**, 9, 712-713 (Readers' Forum), Sept. 1960.

1490. Baron, J. R., and Scott, P. B., The laminar diffusion boundary layer with external flow field pressure gradients, AFOSR TN 59-1268 (Mass. Inst. Technol., Nav. Supersonic Lab. TR 419), 67 pp., Dec. 1959.

Numerical solutions are found for boundary-layer flows with surface injection of a foreign gas. Free-stream pressure gradients for a family of wedge flows are used as in the Falkner-Skan case. Sample results are given for air and for helium injection at the boundary of an air stream flowing at small Mach numbers. Constant temperature boundary conditions are assumed. The injection mass flux must follow special power laws in distance in order to preserve similarity. Transport properties of binary mixtures were determined from Enskog's formulas.

The results are given for favorable pressure gradients only. Heat-transfer reduction by blowing is rather sensitive to negative pressure gradient. With greater values of this gradient, the advantages of helium injection over air diminish. Interesting cases of velocity overshoot exist as in certain hot wall boundary layers without blowing. Boundary-layer blowoff occurs above a critical value of the blowing parameter. This phenomenon seems very sensitive to pressure gradient; however, the blowoff region was not investigated in detail because of computing difficulties, probably arising from infinite boundary-layer thicknesses.

A. Q. Eschenroeder, USA

1491. Gault, D. E., An experimental investigation of boundary-layer control for drag reduction of a swept-wing section at low speed and high Reynolds numbers, NASA TN D-320, 18 pp., Oct. 1960.

An investigation of laminar boundary-layer control by suction for purposes of drag reduction at low speed and high Reynolds numbers has been conducted in the Ames 12-foot pressure wind tunnel. The model was a 72.96-inch-chord wing panel, swept back 30°, which was installed between end plates to approximate a wing of infinite span. The airfoil section employed was a modified NACA 66-012 in the streamwise direction. Tests were limited to controlling the flow over only the upper surface of the model. Seventeen individually controllable suction chambers were provided below the surface to induce flow through 93 spanwise slots in the surface between the 0.0052- and 0.97-chord stations.

Tests were made at angles of attack of 0°, ±1.0°, ±1.5°, and −2.0° for Reynolds numbers from approximately 1.5×10^6 to 4.0×10^6 per foot. In general, essentially full-chord laminar flow was obtained for all conditions with small suction quantities. Minimum profile-drag coefficients of about 0.0005 to 0.0006 were obtained for the slotted surface at maximum values of the Reynolds number; these values include the power required to induce suction as an equivalent drag.

From author's summary

1492. Kirkwood, T. F., An approximate investigation of the effect of boundary-layer-control pumping on powerplant performance, *J. Aerospace Sci.* **27,** 10, 799-800 (Readers' Forum), Oct. 1960.

1493. Roy, A. K., Estimation of the critical viscous sublayer in shock wave boundary layer interaction (in English), *ZAMP* **10,** 1, 82-89, Jan. 1959.

Paper uses similarity considerations to examine the thickness of the viscous sublayer in Lighthill's theory for the interaction between shock waves and boundary layers. For turbulent boundary layers, author concludes that the viscous sublayer is thin and lies well inside the laminar sublayer.

D. W. Holder, England

1494. Li, T. Y., Recent advances in nonequilibrium flow in gas dynamics, AFOSR TN 60-597 (Rensselaer Polytech. Inst., Dept. Aero. Engng. TR-6001), 58 pp., May 1960.

1495. Mull, H. R., and Algranti, J. S., Flight measurement of wall-pressure fluctuations and boundary-layer turbulence, NASA TN D-280, 26 pp., Oct. 1960.

Results are presented for a flight test program using a fighter-type jet aircraft flying at pressure altitudes of 10,000, 20,000, and 30,000 feet at Mach numbers from 0.3 to 0.8. Specially designed apparatus was used to measure and record the output of microphones and hot-wire anemometers mounted on the forward-fuselage section and wing of the airplane. Mean-velocity profiles in the boundary layers were obtained from total-pressure measurements.

The ratio of the root-mean-square fluctuating wall pressure to the free-stream dynamic pressure is presented as a function of Reynolds number and Mach number. The longitudinal component of the turbulent-velocity fluctuations was measured, and the turbulence-intensity profiles are presented for the wing and forward-fuselage section.

In general, the results are in agreement with wind-tunnel measurements which have been reported in the literature. For example, the variation of $\sqrt{\overline{p^2}}/q$ ($\sqrt{\overline{p^2}}$ is the root mean square of the wall-pressure fluctuation, and q is the free-stream dynamic pressure) with Reynolds number was found to be essentially constant for the forward-fuselage-section boundary layer, while variations at the wing station were probably unduly affected by the microphone diameter (5/8 in.), which was large compared with the boundary-layer thickness.

From authors' summary

Symposium on mechanics of real fluids, Proceedings Part 2, Indian Institute of Technology, Kharagpur, India, April 1958; *J. Sci. Engng. Res., India* **4,** 1, Jan. 1960. (Revs. 1496-1514)

1496. Seth, B. R., Boundary layer thickness, 1-6.

1497. Wadhwa, Y. D., Unsteady boundary layers, 7-22.

1498. Roy, A. K., Boundary layers and up-stream influence, 23-38.

1499. Sen, S. P., Heat transfer through the boundary layer of annuli, 39-52.

1500. Bose, N. K., Sediment transportation as a boundary layer problem, 53-66.

1501. Roy, N., A note on sediment transportation as a boundary layer problem, 67-76.

1502. Rao, J. V., Hydraulic model techniques in the solution of civil engineering problems, 77-90.

1503. Narasimhan, M. N. L., Mechanics of flow of real fluids through flexible tubes, 91-104.

1504. Mokadam, R. G., Non-reversible thermodynamics of fluid flow, 105-122.

1505. Bandyopadhyay, G., Some transformations of the equations of compressible flow with heat transfer, 123-130.

1506. Advani, R. M., Application of dimensional analysis and model theory to hydraulic problems, 131-134.

1507. Apte, A. S., Oscillation of water surface inside model harbours and analytical methods of its prediction, 135-142.

1508. Venkateswarlu, D., Applications of model science to some fluid flow problems in chemical engineering, 143-154.

1509. Gayen, A. K., Statistical methods in turbulence studies, 155-189.

1510. Advani, R. M., Hydraulic analysis of complex pipe network by electric network analyser, 190-196.

1511. Panikar, J. T., Experimental study on boundary layer control by suction, 197-216.

1512. Manohar, M., Stability of flow near an oscillating surface, 217-238.

1513. Rao, M. N., Flow pattern and mechanism of atomization in pressure nozzles, 239-250.

1514. Rao, J. V., On the problems of flow of water through porous media and methods of their solutions, 251-264.

End of Symposium

Turbulence

(See also Revs. 1445, 1495, 1509, 1575, 1600)

1515. Lin, C. C., On a theory of dispersion by continuous movements, Part 1; Part 2:, Stationary anisotropic processes, *Proc. Nat. Acad. Sci.* **46,** 4, 566–570, Apr. 1960; **46,** 8, 1147–1150, Aug. 1960.

These papers deal principally with the relative dispersion of two tagged points in a continuum by the action of turbulence. Velocity differences at two times in the history of a single tagged point are also discussed. The turbulent continuum is assumed to be homogeneous. In Part I it is assumed to be isotropic also, while in Part II this restriction is not made. It is assumed that initial separation and relative velocity are negligible compared to these quantities at the time of observation. Two fundamental assumptions are made: (1) the relative acceleration is assumed stationary, (2) the autocorrelation of the relative acceleration, $R(\tau)$, is assumed to have a behavior such that its integral $\int_0^{\tau_1} R(\tau)\,d\tau$ attains an essentially stationary non-zero value as τ_1 increases, dropping to zero (a kinematical requirement) only for very large τ_1.

Using these assumptions, results are obtained similar to those of Obukhov (who assumed a Markov process) and others who used the Kolmogoroff theory. In particular, Richardson's law of dispersion is obtained, together with an extension of this law to anisotropic fields.

Assumptions (1) and (2) are made largely without physical justification. Reviewer feels that such justification is not straightforward. Since the problem is essentially kinematical, reviewer feels that author's statement that (1) asserts "that the pressure forces acting on a fluid particle are essentially independent of its velocity" is questionable. Reviewer feels that the justification for (1) and (2) should relate initial separation, and separation at the observation time, to the micro- and macro-scales of the turbulence, and consequently relate these scales to each other. Such relations might clarify the connection of (1) and (2) to the Kolmogoroff theory.

J. L. Lumley, USA

1516. Mikhail, S., Mixing of coaxial streams inside a closed conduit, *J. Mech. Engng. Sci., Lond.* **2,** 1, 59–68, Mar. 1960.

The equations of motion are written in integral form and the change in total momentum flux and total pressure forces between two adjacent transverse sections of the streams is equated to the turbulent shear acting on the stream between these sections. The following assumptions are made: (1) the velocity profile in dimensionless form is identical for all values of x in each region of the different mixing regions and (2) the axial velocity profile in the mixing zone may be represented by a cosine curve. In order to determine the turbulent shear stress, the form of the momentum transfer theory of turbulent flow is adopted, and with this assumption the entire velocity field is worked out. The second part of the paper describes experimental data obtained from a coaxial air-to-air jet pump having a mixing chamber of constant diameter. The main conclusion to be drawn is that the theory gives an over-all picture of the mixing process which is in the main correct, though it might be inaccurate in some of the details.

D. Gh. Ionescu, Roumania

Aerodynamics

(See also Revs. 1439, 1453, 1455, 1464, 1467, 1472, 1486, 1491, 1531, 1545, 1560, 1571, 1572, 1573, 1636, 1644, 1659, 1660, 1664, 1665, 1670, 1671, 1676, 1678, 1690)

1517. Kamimoto, G., Ishida, S., and Akamatsu, T., An approximate theory of airfoil cascade in subsonic flow, *Bull. JSME* **3,** 10, 177–184, May 1960.

Linearized distributions of vortices and sources are used to replace the airfoil cascade and the appropriate flow parameters are derived by summation. The gap chord ratio r is assumed to be large, and approximate results are obtained by expanding in a power series in $1/r$. Reviewer believes that the standard method (not involving the restriction on r) of transforming a cascade into a single airfoil and then applying thin-airfoil theory is superior to the method given in the paper and involves no greater computational effort.

L. C. Woods, England

1518. Ruetenik, J. R., Traveling-gust method for airfoils in strong blast waves, *J. Aerospace Sci.* **27,** 10, 795–796 (Readers' Forum), Oct. 1960.

1519. Chard, K. F. S., and Deakin, J., A wind tunnel investigation into the factors affecting the spanwise load distribution over the vertical tail surface of an aircraft having a wedge-type rear fuselage, Aero. Res. Counc. Lond. Curr. Pap. 491, 29 pp., 1960.

The spanwise load distribution over the vertical tail in yaw has been investigated with a wind-tunnel model of a typical low-speed, straight-winged aircraft without propeller, having a wedge-type rear fuselage. The tests revealed considerable variation in the direction of flow in the region of the vertical tail and subsequently an appreciable effect on the spanwise load distribution. In the appendix an approximate estimate of spanwise load distribution on the vertical tail in a yawed condition is given. The discrepancy between the measured load and that predicted from the assumed vortex system is attributed by A. R. Collar (in Appendix II) to the use of a finite number of horseshoe vortices.

W. Wuest, Germany

1520. Korbacher, G. K., and Sridhar, K., A note on the induced drag of jet-flapped wings, *J. Roy. Aero. Soc.* **64,** 593, 290–291 (Tech. Notes), May 1960.

Authors explain apparent discrepancy in change of "effective aspect ratio" (as obtained by computing relation between induced drag and lift for a jet-flapped wing of finite span) between what would be expected on "intuitive" grounds and what is actually obtained both experimentally and theoretically. The "intuitive" expectation is based on mechanical flap analogy, on basis of which one expects decreased aspect ratio for increased blowing. Authors show that "intuitive" approach fails in not taking account of additional lift created by vertical component of jet momentum far behind wing. In fact, if induced drag is computed in terms of circulation lift only (leaving out jet momentum lift) the result does indicate a reduction in effective aspect ratio, in agreement with "intuitive" expectation. Nevertheless, the truly useful results are in terms of total lift and total induced drag; these lead to an *increase* in effective aspect ratio, "intuition" notwithstanding.

J. E. McCune, USA

1521. Yen, K. T., On the thrust hypotheses for the jet flap including jet-mixing effects, *J. Aero/Space Sci.* **27,** 8, 607–614, Aug. 1960.

For incompressible potential flow, the problem of evaluating thrust of a jet flap is shown to correspond to the thickness problem of thin airfoil theory. The jet angle does not enter in the linearized analysis so a linear thrust hypothesis is proposed. The momentum-integral method is then used to show how higher-order terms modify the linear result. The mixing phenomenon is assumed to be governed by boundary-layer-type equations. Approximate formulas relating the thrust and the jet angle are derived.

G. E. Nitzberg, USA

1522. Graham, M. E., and Lagerstrom, P. A., Reduction of minimum-drag problems in supersonic flow to two-dimensional potential problems by means of the combined flow field, Douglas Aircr. Co. Rep. SM-23901, 101 pp., Mar. 1960.

Authors report the development of the problems of minimum drag of three-dimensional planar and nonplanar wings or body in supersonic flow. They use methods which differ from those used previously by other authors. Especially the authors paid attention to the minimum drag problems which include the constraint of a specified lateral distribution of base area. They give a new formulation of these problems. In particular the equations for steady inviscid flow with sources and impressed forces are derived. The relations between the body force, body volume, body base area, and the singularities are found. Also the combined flow equations and equations for the total drag and for the interference drag are given. It is shown how to find the total singularity strength from the combined flow field. Authors give the conditions imposed upon the combined flow field to assure optimum singularity distribution under the constraints of (1) total lift specified, (2) total lift, total pitching moment, and lateral distribution of base area specified, (3) total base area specified, and (4) total volume and lateral distribution of base area specified.

T. Riabokin, USA

1523. Tan, H. S., On a special Bolza variational problem, and the minimization of superaerodynamic hypersonic nose drag, *Quart. Appl. Math.* **17**, 3, 311-314 (Notes), Oct. 1959.

The search for optimum nose curve $y(x)$ that minimizes the superaerodynamic hypersonic drag of an axially symmetric body leads to a variational problem. Drag can be presented as a sum of a function of known nose parameters which are the base radius and tip radius, and of an integral whose value depends on the shape of optimum function $y(x)$ which minimizes the drag. This is a special case of the Bolza problem. An optimum solution must be that of Euler's variational equation. The limitations on the admissible values are discussed.

P. Bielkowicz, USA

1524. Buthod, A. P., and Tien, C., Critical velocity of gas, *Canad. J. Chem. Engng.* **38**, 3, 74-77, June 1960.

Authors derive thermodynamic relationships for calculating the sonic velocity in a gas from its critical temperature T_c, critical pressure P_c, and heat capacity, via the reduced equation of state (compressibility factor Z as function of T/T_c and P/P_c.) Generalized graphs are presented of those derivatives of Z which occur in the relationship. Comparison between observed and computed sonic velocities of carbon dioxide and ethylene shows fairly good agreement.

Reviewer protests use in this paper of term "critical velocity" to indicate propagation velocity of sound, because of confusion with "critical" PVT phenomena and "critical velocity" for transition from laminar to turbulent flow. Numerous typographical errors in equations are coupled with inadequate definition of symbols employed to cause further confusion.

I. M. Krieger, USA

1525. Tobak, M., and Lessing, H. C., Study of the aerodynamic forces and moments on bodies of revolution in combined pitching and yawing motions, NASA TN D-316, 24 pp., May 1960.

Errors are pointed out in the commonly accepted derivation of the equations of the aerodynamic forces acting on bodies of revolution during pitching and yawing flight. A more correct derivation, in which forces are resolved in and perpendicular to the plane of the angle of attack, is given, which seemingly corrects the errors inherent in past methods. The aerodynamic forces derived by authors' method are nonlinear although they are expressed in terms of linear aerodynamic coefficients or stability derivatives.

Also, certain criticisms are made concerning current methods of calculating magnus forces. It is pointed out that the magnus force of a yawing and pitching body of revolution need not be zero when the rolling velocity is zero.

E. K. Parks, USA

1526. Vedrov, V. S., Romanov, G. L., and Surina, V. N., The airplane as an object of control (Block diagrams of the equations of perturbed airplane motion), NASA TT F-5, 52 pp., Oct. 1959.

The disturbed motion of an aircraft is represented with the aid of single-loop block diagrams, which permit one to apply readily the methods of the automatic control theory to stability problems. The considered equation systems are presented in such a form that the links of the block diagrams should be of the first or second order and the signals at the input and output of the links should have a definite physical meaning.

Both the longitudinal and the lateral motions are discussed. The analysis of the basic characteristics of the links is made on the basis of a numerical example concerning a conventional subsonic jet aircraft. The method is verified by finding in a simple way some known results (e.g. stability deterioration during the climb, etc.). Some new aspects are also presented. It is analytically deduced that at high speeds an artificial damping of the long-period motions is necessary; the dependence of the aircraft angular deviation on the elevator setting is analyzed by using as a parameter the frequency range of the respective type of motion. As concerns the lateral motion, the possibility is indicated of obtaining abbreviated equations for each type of motion.

Paper is of real practical interest as it supplies the theory of aircraft dynamic stability with more up-to-date techniques. The authors' effort to stress the influence of various kinematic and design parameters on the disturbed motion by analyzing the expressions of transfer functions is particularly useful. It is regrettable that the English translation does not indicate the date the original report was issued, therefore one is unable to situate it in time as compared to other papers with a similar objective (see for instance AMR **12** (1959), Rev. 328).

T. Hacker, Roumania

1527. Aerodynamics of harvesting, *Engineering* **190**, 4923, p. 293, Aug. 1960.

Vibration and Wave Motion in Fluids

(See also Revs. 1382, 1436, 1507, 1524, 1555, 1565, 1652, 1715, 1716)

1528. Rabinovich, B. I., On small harmonic oscillations of a cylindrical shell along the axis of which an ideal gas flows with supersonic velocity, *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **23**, 5, 1255-1262, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

A system of uniformly distributed supersonic sources is located at the flat bottom of a circular cylinder. The supersonic stream of the ideal gas originating at the bottom leaves the cylinder at the other end. The gasdynamic forces acting at the walls and bottom of the cylinder as a result of small forced oscillations of the cylinder are investigated. The modes considered are a harmonic translation in a direction perpendicular to the axis of the cylinder and a harmonic rotation about an axis, also perpendicular to the cylinder axis. Linearized theory is used. The result is that the unsteady effects due to the compressibility of the gas are negligible if $(k/1.844)^2 \ll 1$, where $k = \omega R/a$ is the reduced frequency, referred to the frequency of oscillation ω, the radius R of the cylinder and the speed of sound a in the gas. If this condition is

satisfied, the gas stream may be considered as absolutely stiff in the transverse direction. For larger values of the reduced frequency the unsteady effects, dependent upon the Mach number of the stream, can be obtained from formulas given in the paper. The problem is of importance for rocket propulsion.

A. I. van de Vooren, Holland

1529. Roshko, A., and Rubinstein, M., Tables of flow functions for a simple non-steady expansion wave, Douglas Aircr. Co. Rep. ES 40031, 58 pp., Sept. 1960.

The nature of a nonsteady expansion wave and its equations are reviewed. Tables and charts of twelve functions of a nonsteady expansion wave are presented for both $\gamma = 7/5$ and $5/3$, using the dimensionless flow speed as the independent parameter.

From authors' summary

1530. Stepanchuk, V. F., and Khutskii, G. I., Evaluation of the resistance of inertia in gas conduits (in Russian), *Izv. Vyssh. Uchebn. Zavedenii, Energetika* no. 5, 88-90, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 4866.

The flow of gas in an extensive pipe system is linked with the appearance of wave resistance due to the presence of forces of inertia which develop as the velocity of the gas motion accelerates. An evaluation is given for the magnitude of the inertia losses of pressure and an investigation is made of the influences of these losses on the throughput capacity of the pipe system. It was shown that inertia resistance results in a reduction in throughput of gas in the conduits. It was explained that the influence of inertia resistance increases with the increase in the fall of pressure in the pipe system and with a decrease in the relative length of the latter. The influence of inertia resistance on the throughput of gas is not large in cases of widespread conduit systems with relatively small falls in pressure (of the order of 3 to 5). In this case the throughput of gas can be determined with sufficient accuracy by means of the Weymouth formula.

Yu. A. Lashkov

Courtesy Referativnyi Zhurnal, USSR

1531. Prokof'ev, V. A., Absorption and dispersion of weak forced waves of very small and very large frequency under the influence of heat transfer by radiation (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 12, 15-23, Dec. 1958.

Present paper is a sequel to the article published in *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 7, 1957, where it was shown that dispersion of weak waves is described by a bi-cubic equation, if viscosity, heat conductivity and radiation heat transfer are considered. If radiation is totally neglected or if it follows Newton's law only two kinds of waves can be excited; if the absorption of radiation is taken into account a third type may arise, corresponding to the third pair of solutions of basic equation. All three types arise simultaneously and are difficult to detect separately, but their damping coefficients are not equal, hence at some distance from the origin only one type of waves will still exert detectable action. Difference in the propagation velocity may also be used it creating a method for their recording. Particular cases are examined in the present paper.

P. Bielkowicz, USA

Book—1532. Loginov, V. N., Determination of wave formations when designing a harbour (in Russian), Moscow, Marine Transport, 1958, 185 pp. + illus. 9 r 30 k; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5006.

The first paragraph of this monograph indicates the importance of investigations of waves when designing a harbour. Examples are given (the port of Madras) where, in consequence of the faulty positioning of the protective installations, the harbor was affected by silt even during the period of construction. It is shown that such faulty location of the protective installations can also result in the deterioration of the ships' berths. A description is given of the "Tiaguna" phenomenon. Descriptions are given in the second paragraph of the fundamental factors determining the wave regime in the waters adjacent to the port and of the investigational methods. Mention is made of the predominant value of wave diffraction. When referring to investigational methods the author comments on the unsatisfactory level of development of the theory and the impossibility of attaining a sufficiently profound basis for the study of the question by the use of empirical methods.

A review of the theoretical and semiempirical methods of investigating the diffraction of waves forms the subject of the third paragraph. There is a comparative appraisal of the different methods and the limits of their application are indicated. The unsatisfactory nature of the generally used hypothesis regarding the even propagation of energy along the front of the diffracted wave is referred to (the possible error in calculating the amplitude may be of the order of 200 to 300%). The fourth paragraph is devoted to making an inventory of the materials required to carry out the design, the procedure for their determination and the general sequence adopted for the calculations. The method used for determining the parameters of wind waves is the subject of paragraph 5. It commences with an exposition of the elements of the theory of prognosis of winds. This is followed by a short survey of the methods of calculation used for wind waves in accordance with the wind's parameters. A recommendation is made to use Yu. M. Krylov's method. In the sixth paragraph the phenomenon is described of deformation of wind waves in the "zone of damping" and nomograms are provided for the calculation of the relative increase in the length of the wave and of the period and relative decrease in the amplitude.

The deformation of waves in shoal water is the subject dealt with in the seventh paragraph. Special attention has been given to the appearance of refraction (distortion of the front of the wave in consequence of variation of depth along the front). The theoretical work done on the subject is scrutinized and the conclusion reached that the sole reliable method appears to be the graphical method for determining the distortion of the wave's front, using a chart of assigned depths with the former. A detailed account of these methods is furnished as well as examples of the calculations. Translating (migrating) and "breaker" waves are described. It is demonstrated that the calculations for such waves can be carried out with a large degree of precision by following the theory of a solitary wave. The calculations given in these paragraphs are used for the evaluation of the parameters of waves at the harbor gates.

The general sequence of calculations for the parameter of waves in the seas near the port forms the subject of paragraph 8. Paragraph 9 reviews papers devoted to the appearance of diffraction of waves in the gates of the port. A large number of photographs are reproduced and also diagrams of the waves diffracting at the gates of the port. A new method of calculation is proposed based on the circumstance that the front of the diffracting waves practically coincides with the line of equal potential in a conforming image of the region of water adjacent to the port on a semi-strip. It is demonstrated that this method, in comparison with the method previously used, enables more precise results to be obtained. A number of numerical examples are published in support.

The tenth paragraph is a collection of data for use in engineering calculations for diffraction, covering a variety of cases. There is also a large number of empirical formulas; the extent of their application is indicated. Sets of nomograms and illustrations are also furnished. Paragraph 11 is devoted to descriptions of calculation procedures for refraction of waves diffracted at the gates of the port. A graphical method of calculation is described; this appears to be the method of refraction in shoal waters given in the seventh paragraph but altered in form. In the twelfth paragraph

graphical methods are given (illustrated by examples) for the calculation of interference and reflection of waves (from the shores and lateral surfaces of protective structures).

The thirteenth paragraph contains recommendations for the rational grouping of harbor structures. The last three paragraphs deal with recommendations for, and methods to be used in, investigations in laboratories with three-dimensional models of the ports.

N. N. Moiseev
Courtesy Referativnyi Zhurnal, USSR

1533. Gaillard, P., Non-linear oscillations of harbour waters (in French), *Houille Blanche* **15**, 2, 164–172, Mar./Apr. 1960.

Author considers mathematically the generation of swell in a rectangular basin due to a spectrum of first-order oscillations at an opening. It is shown that the inclusion of second-order terms results in short-period oscillations generating long-period swell, which is a danger to shipping. Two waves of periods T_1 and T_2, for example, generate oscillations of periods $T_1T_2/(T_1 \pm T_2)$. Also considered are analytical problems of the Fourier-Bessel representation of waves produced by model wave-makers.

J. N. Hunt, England

1534. Afonskiy, A. V., Wave pressure on single piers (in Russian), *Gidrotekh. Stroit.* **28**, 5, 39–40, May 1959.

Empirical formulas for determination of wave pressure on single piers of various cross sections and heights not exceeding 8–10m are given. The results are based on model studies where wave pressure was measured at points along the height and the periphery of the piers.

An example gives the comparison of results by the various methods available. It is claimed the presented technique is most accurate for the type of piers considered. The results obtained by this technique are shown to be in good agreement with at least one other method used previously. V. Chobotov, USA

1535. Hofsommer, D. J., Free oscillations in a rotating semicircular bay (in English), Stichting Mathematisch Centrum, Amsterdam, Rep. TW 47, 16 pp., Dec. 1958.

Free oscillations on a shallow semicircular bay of constant depth are studied by means of Lauwerier's theory of trigonometric series with prescribed phases. Friction is neglected. A determinantal equation results which implicitly determines free frequencies of semicircular bay as a function of Coriolis coefficient. An explicit formula is derived for a first approximation in the square of ratio of Coriolis coefficient and frequency. Lamb's treatment of circular lake is included for comparison.

From author's summary by A. Balloffet, USA

1536. LeMehaute, B., and Brebner, A., An introduction to the mathematical theories of two-dimensional periodic progressive gravity waves, Queen's Univ., Ontario, Civ. Engng. Dept. Rep. no. 11, 49 pp., July 1960.

Report deals with the fundamental hydrodynamics leading to the standard relationships for the identities met with in two-dimensional wave studies. The relationships for velocity, breaking, mass transportation, etc., are introduced and discussed. This report forms the basis for subsequent reports using such wave theory.

From authors' summary

1537. Legen'kov, A. P., Plane free vibrations of an ideal homogeneous liquid in an infinite channel of variable section (in Russian), *Izv. Akad. Nauk SSSR, Ser. Geofiz.* no. 8, 989–994, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 4985.

The paper is a study of the propagation of long progressive waves in a channel of variable section. Special cases are investigated in which the bottom and the width are constant everywhere, with the exception of some portion where they change in accordance with the linear principle. The author derives exact solutions for these problems in Bessel functions and analyzes the results obtained. He claims that he was successful in obtaining a solution which refutes Green's limiting conditions, which in essence state that the parameters of the channel should vary only slightly over the extent of one wavelength. But the equations for long waves also only hold good in cases where the parameter changes of the channel proceed slowly. In a case of this type these equations (subject to the condition that the depth is small) take on an asymptotic character. Should the curvature of the line happen to be large it would not be possible to disregard it in general, even for the vertical accelerations. Consequently the analysis carried out in the paper under review is based on faulty premises.

N. N. Moiseev
Courtesy Referativnyi Zhurnal, USSR

1538. Shutilov, V. A., Optical investigations of a large-amplitude ultrasonic wave form in a liquid, *Soviet Phys.-Acoustics* **5**, 2, 230–238, Nov. 1959. (Translation of *Akust. Zh., USSR* **5**, 2, 231–241, Apr./June 1959 by Amer. Inst. Phys., New York, N.Y.)

It is demonstrated in the present paper, on the basis of the distribution of light intensity in a diffraction pattern obtained upon diffraction of the light by ultrasonic waves of large amplitude, that the form of the ultrasonic wave can be determined and its distortion during the process of propagation in the liquid can be traced. Experimental graphs showing the dependence of the wave distortion coefficient on the potential on the radiating quartz and the distance from the sound source are illustrated and evaluated.

The possible application of the connection between the wave form and the light intensity distribution in the diffraction pattern for determining the absolute values of the intensity of large-amplitude ultrasonic waves in a liquid is indicated.

J. S. Arnold, USA

1539. Ter-Krikorov, A. M., An exact solution of the problem on the motion of a vortex under the surface of a liquid (in Russian), *Izv. Akad. Nauk SSSR, Ser. Matem.* **22**, 2, 177–200, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 4979.

A flow of heavy liquid with a depth b meets an immovable vortex spot with a determined circulation. The presence of this vortex causes deformation of the free surface of the flow and gives it a wavelike form. Great difficulties are encountered in the determination of the form of this surface when accuracy is insisted on in the computation of all the boundary conditions of the problem, which have to be taken into account with the surface of the wave an unknown. The present paper puts forward a method for the derivation of the exact solution mentioned and, what is of principal importance, the convergence is shown of that infinite, iterational process whose solution is brought about, in regard to the construction of the flow, by the method advocated by the author. The author derives a nonlinear integral equation for the boundary values on the contour of the strip for the imaginary and actual portions of the logarithm of the product of the analytical function establishing the said conforming image; to do so he adopts the method usually accepted in such problems of a conforming image of the region of flow, bounded by the bottom and an unknown free surface on an infinite strip. This equation is investigaged by the methods in the theory of nonlinear operators and proof is furnished for the existence of the solution and its uniqueness for the velocity of flow c, exceeding the critical velocity $\sqrt{gb}$; here it is assumed that the circulation of the vortex is sufficiently small in comparison with the magnitude cb.

L. N. Sretenskii
Courtesy Referativnyi Zhurnal, USSR

1540. Kiladze, R. M., Determination of the parameters of a pressure wave with consideration for the curvature of the streams (in Russian), *Soobshch. Akad. Nauk GruzSSR* **20**, 2, 143–150, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 4991.

A plane flow of a heavy liquid moving along a horizontal bottom is brought to a standstill by the sudden submersion of a grid. The relation is found of the maximum height of the wave to its mean level by making use of known hydraulic formulas and of experiments specially made in a tank. An investigation is carried out to ascertain the criteria for the break-up of the wave and an empirical curve was obtained for the determination of the length of the formation portion of the wave. M. I. Gurevich

Courtesy Referativnyi Zhurnal, USSR

1541. Rodzievskii, V. A., Frequency and damping of the natural vibrations in linear vibrotransmitters of the inertial type with liquid damping (in Russian), *Dop. Akad. Nauk URSR* no. 7, 716–720, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 4710.

Results are given of an investigation to determine the influence exercised by the connected mass of the damping liquid on the frequency and damping of the natural vibrations of vibrotransmitters. M. E. Temchenko

Courtesy Referativnyi Zhurnal, USSR

Fluid Machinery

(See also Revs. 1517, 1571, 1576, 1604, 1608, 1623)

Book—1542. Jankowski, F., Lazarkiewicz, S., and Troskolanski, A. T., Pumps in sanitary engineering [Pompy w technice sanitarnej], Warszawa, Arkady, 1960, xx + 500 pp.

A comprehensive handbook on pumps for water supply and sewerage contains theoretical fundamentals, classification and construction of all types of pumps, their selection, design, installation and operation. Acceptance tests and automation conclude the content. Tables include important data on (Polish) standards. Bibliography is presented at the end of each chapter and covers European, American, and Russian literature. Figures and print are of high quality. S. Kolupaila, USA

1543. Stepanov, E. I., and Fel'dman, L. P., The hydraulic calculations for deep-seated diaphragm-fitted pumps (in Russian), *Nauchn. Trudf Novocherk. Politekhn. In-ta* **67**, 81, 91–100, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 3130.

A method is described for the hydraulic calculations of the regular process of work of deep-seated diaphragm-fitted pumps, designed for pumping water out of deep wells (of the order of 200m and more). The proposed method is applicable to the calculations for the above type of pumps fitted with rubber pulsators and also when fitted with pistons in the upper part of the apparatus. Correlations are obtained determining the distribution of pressure in the piping system. Conditions are found in which a break in the column of liquid from the upper working unit is possible (piston or pulsator) and the consequent appearance of resonance. A hydraulic calculation was carried out, based on the proposed method, using an experimental sample of a deep-seated diaphragm-fitted pump. The results of the calculations agreed most satisfactorily with the experimental findings. No investigation was made in this study of the starting regime and of the influence of the column of liquid in the delivery tube-conduit on the pump's characteristics. Yu. A. Lashkov

Courtesy Referativnyi Zhurnal, USSR

Book—1544. Petermann, H., Design and elements of fluid machinery [Konstruktionen und Bauelemente von Stromungsmaschinen], Berlin, Springer-Verlag, 1960, 76 pp. DM 12. (Paperbound)

This is a valuable work for students of fluid machinery at technical institutions and technical universities, where it is of great importance to have some practice in the design and construction of such machinery. The work offers a collection of structural examples, with drawings and illustrations of complete radial- and axial-flow blowers, pumps, compressors and gas-, water- or steam-turbines, as well as detailed drawings of their components.

The illustrations serve as typical examples of modern and, in some cases, less recent designs from well-known firms all over the world, with explanatory notes accompanying the figures where necessary. The following groups are included: impellers, rotors and runners; guide vanes, diffusers, etc.; blade manufacture; balancing axial thrust; cooling and heating in fluid machinery; casings, housings and associated components; shaft packings and seals. The illustrations are of the usual high quality expected from this publisher. S. I. Wiselius, Holland

Book—1545. Vavra, M. H., Aero-thermodynamics and flow in turbomachines, New York, John Wiley & Sons, Inc., 1960, xvi + 609 pp. $14.50.

This advanced text is divided in three parts. In part 1 (103 pp) the fundamental relations for the analysis of arbitrary flows are derived from general principles of fluid mechanics and thermodynamics. Among topics covered are kinematics of fluid particles, general stress conditions of fluid particle and equation of motion.

Part 2 (127 pp) discusses flows of more special character, primarily of interest to the flow in turbomachines, such as relative motions and rotational motion.

Part 3 (240 pp) is directly concerned with flows in turbomachines. Topics like axisymmetric flows in rotating and stationary passages, axial compressor stages and two-dimensional potential flow through arbitrary cascades are presented. Book closes with appendices devoted to resumes of vector and dyadic analysis indispensable for understanding the presentation, and tables of results for turbine stages with optimum efficiencies.

The typography and illustrations are very good.

Author treats the subject from a mathematical standpoint, using vector and dyadic analysis, and emphasizes basic physical processes. Test results and empirical formulas are few and methods for establishing performance maps are excluded.

Author has presented matters logically and clearly, which makes the book particularly suitable for individual study. It is an excellent book which can be highly recommended to engineers and graduate students with requisite mathematical background. O. Soderberg, Sweden

Book—1546. Markov, N. M., Investigation of the part of a turbine between the inlet and outlet valves (in Russian), Moscow-Leningrad, Mashgiz, 1958, 127 pp. + illus. 4 r; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 4941.

The greater part of the book is a duplication of matter previously published by the author, in particular in the book "Calculations for the aerodynamic characteristics of the blade assemblies of turbomachines" (Mashgiz, 1955). By comparison with that publication the new book eliminates large mathematical computations dealing with the calculating of the boundary layer in two-dimensional and three-dimensional flows, and adds some new material, in particular, information concerning the conversion of experimental data to a practical form. The first three chapters are taken up with the exposition of questions known in the theory of turbines, relating to the classification and description of losses in the part of the turbine between the inlet and outlet valves, the formulation of conditions for the modelling of turbines and the influence of Reynolds and Mach numbers, and also with descriptions of typical experimental plant for the investigation of turbine lattices. Chapter IV deals with the experimental characteristics of

controlling and working lattices and of stages as a whole. The effectiveness of a turbine with cylindrical ovals is lower than that of turbines with twisted blades and it can only be used with short blades: $d/D_{mean} \leqslant 1/10$. A comparison is given in Chapter V between the calculated and the experimental values of the efficiency (k.p.d.) of stages of various types. The comparison shows that in the majority of cases the difference between the calculated and the experimental values of efficiency (k.p.d.) does not exceed 2 to 3%. Since a series of second-stage losses in such a calculation is not taken into account, it is recommended to lower the calculated value of efficiency (k.p.d.) by 1.5 to 2%. Chapter VI is devoted to the calculation of the output of the working body through the blade ring. Chapter VII gives numerous data from the literature regarding the influence of clearances on the efficiency (k.p.d.) of the stage. In Chapter VIII author evaluates the influence of the thickness and form of the outlet edge on the profile losses in the lattice. Various data taken from the literature are quoted in Chapter IX for the determination of losses due to disk friction and also on account of the partial supporting of the working body. Chapter X gives some data on the aerodynamic characteristics of nozzles and valves.

V. Kh. Abiants

Courtesy Referativnyi Zhurnal, USSR

1547. Broecker, E., On the analytical calculation of the performance of radial flow cascade (in German), *Forsch. Geb. Ing.-Wes.* **26,** 2, 62–67, 1960.

The radial flow blower which is being considered consists of a rotating cascade with blades whose cord is relatively small as compared to the radius. Such an arrangement allows the usage of potential theory as it is used in axial flow compressors. The agreement between the predicted performance and tests is good for accelerated flow (turbines) and near the design point. It is not good when the flow is decelerated and at other than the design point and the author proposes a theory which allows to calculate the pressure increase developed as a function of the flow rate. This theory is based on the observation that the meridianal flow velocities vary over the span of the blades. The work input corresponding to this velocity variation is then calculated and better agreement between experiment and calculated values is obtained.

The additional effects of secondary flow and separated flows are indicated. Ten figures show pressure coefficients as a function of the flow coefficient according to theory with the number of blades as a parameter and the measured values. Also, some work inputs over the blade span for various flow rates are given.

H. P. Eichenberger, USA

1548. Ohashi, H., Investigations of compressor blade cascades of marked curvature in tandem (in German), *Ing.-Arch.* **27,** 4, 201–226, 1959.

In view of the marked increase in losses associated with flow through compressor blade cascades of large curvature, both theoretical and experimental studies were made on cascades of blades set in tandem. The theoretical studies were made on blades of profiles NACA 0010, 4410 and 8410. These studies show that tandem blade cascades have lower losses than single blades over a large region. The analysis leading to this conclusion was carried out on the assumption that the two rows of the tandem arrangement were separated by sufficient distance so that there was no interaction by one row on the flow passing through the other.

An experimental investigation was made of a tandem cascade of blades to profile NACA 8410. When the two rows were separated by a large distance, a total curvature of 50° was achieved and the total losses were in good agreement with those analytically determined. The measurements show that the flow losses could be reduced below those corresponding to large separation by bringing the two blade rows together to a separation equal to 15–50% of the blade spacing, and total losses have a minimum for a specific relation of the leading edge of blades of the second row to the extension of the chords of those of the first.

Analysis by potential theory did not explain the effect of interference on the losses from the second row of blades.

M. A. Mayers, USA

1549. Smigielski, J., and Puzyrewski, R., Some results of experimental investigation of a cascade of guiding turbine vanes of small height (in Polish), *Arch. Budown Maszyn* **6,** 3, 363–380, 1959.

Cascades of guiding turbine vanes of height-to-width ratio varying between 0.38 and 0.97 were tested at low Mach numbers. The measurements concerned outlet angles, losses and effect of cascade pitch. It was found that the outlet angle is smaller than the theoretically predicted one and that, generally, losses are smaller than in acting type cascades. Conclusions concerning other investigated parameters are given.

A general discussion on the accuracy of the measurements and the range of applicability of the results obtained is missing.

W. Fiszdon, Poland

1550. Kull', A. M., Some results obtained in investigating an active lattice of turbine blades when these showed little divergence in their geometrical form (in Russian), *Trudi Leningrad Politekhn. In-ta* no. 193, 150–156, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 4870.

Some results are given of an investigation to ascertain the influence exercised by the precision of the set-up of the lattice profile and the influence of some local divergences in the geometrical form of the profile on the aerodynamic characteristics of the lattice. Experiments show that in the case of the lattices tested the reaction due to the twist of a single blade within the limits of −2.5° to 2.5° only affects two adjacent blades. Because of this it is expedient to use the characteristics of the loss and the angle of flow, averaged for three spacings. These characteristics made it apparent that a deviation in the angle of the set-up of the profile to the extent of 2.5°, 1° and −1° has practically no influence on the profile losses. However, with a magnitude for this deviation of −2.5° (the twist of the blade is considered to be negative when there is an increase in the outlet angle of the flow) a sharp increase in profile losses (of ~ 2%) is observed, which is explained by the decrease of turbulent effect in the channel in such a case.

The deviation in the geometrical form of the profile was effected by glueing onto the profile a local angular thickening of the order of 0.6 mm (with a chord length for the profile of 52.3 mm) made of dense paper covered with tracing cloth. The edges of the glued section merged smoothly with the profile. Three cases were investigated: (1) the inlet portion of the profile was thickened for ⅓rd of the extent of the curvature of the contour; (2) the outlet section was thickened for ⅓rd of the extent of the curvature of the contour; (3) the whole of the profile was treated as in (1) and (2). It was shown that in static blow-throughs the maximum diminution in the efficiency of the lattice (k.p.d.) does not exceed 0.5%. However when carrying out investigations on a model turbine the effect of thickening the profile proved to be appreciably larger than in the full-scale plant. When half of the working blades of the outlet portion were thickened the efficiency (k.p.d.) of the stage was 4% lower than the efficiency (k.p.d.) of the inlet stage.

V. Kh. Abiants

Courtesy Referativnyi Zhurnal, USSR

1551. Kriebel, A. R., Seidel, B. S., and Schwind, R. G., Stall propagation in a cascade of airfoils, NASA TR R-61, 51 pp., 1960.

See AMR 13(1960), Rev. 914.

An experimental investigation of stall propagation in a stationary circular cascade in which high-speed schlieren and interferometer photography is used is described. This investigation suggests an analytical approach to the study of stall propagation which is valid only for an isolated blade row in an infinite flow field but which is not restricted to small unsteady perturbations or to an assumed simplified cascade geometry. Conditions necessary for the existence of the assumed type of stall cells are described and equations are derived for the velocity of stall-cell propagation.

From authors' summary

1552. Beaufrere, J., The effect of scale on cavitation in hydraulic turbomachinery (in French), *Houille Blanche* **15,** A, 485–492, July/Aug. 1960.

Author describes an experimental investigation of cavitation phenomena in hydraulic turbomachinery carried out with the intention of reducing the safety factor that has to be allowed on the prototype when using model results.

A theoretical study shows that scale effects affect the absolute pressure, the critical pressure and factors such as the boundary layer and breakaway, which depend upon turbulence. The results of this study show that the Froude similitude is the similitude which is associated with the smallest scale effect for horizontal shaft machines and that it can practically be neglected provided that turbulence in the model is not too small, i.e. provided that the scale of the model is not too small. Any remaining scale effects make for greater safety since cavitation occurs more easily in the model than in the prototype, especially since the model head is smaller.

The results of this study were checked by three series of experimental tests.

From author's summary

1553. Voitashevskii, D. A., The mean velocities of the flow in an axial hydroturbine (in Russian), *Trudī Vses. N.-i. In-ta, Gidromashinostr.* no. 21, 3–18, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5116.

The linkage is established between the parameters of a three-dimensional flow in an axial hydroturbine obtained previously by the author [title source no. 16, 1953, and others] and the parameters of some equivalent wheel of the mean lattice of the profiles. In securing this result the author has deemed it possible to assume a change of circulation created by the blades for blades of arbitrary form, and of pressure up to and beyond the wheel, independently of the radius. The radius corresponding to the equivalent section obtained differs only slightly from the radius for the mean section.

N. A. Kolokol'tsov

Courtesy Referativnyi Zhurnal, USSR

1554. Ivanov, V. I., Investigation of the flows in an axial hydroturbine when the engine is racing (in Russian), *Trudī Vses. Nauk-i. In-ta Gidromashinostr.* no. 21, 19–38, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5117.

This is a continuation of previous work [see title source no. 18, 32–51, 1954]. Formulas are established, with the aid of the basic equation for a turbine and of the equation for an evenly rotating lattice, in order to calculate the parameter of racing $\psi = nD^3/60Q$, the number of revolutions n and input Q in the racing regime of the hydroturbine while taking into account the mechanical loss, and the conditions are determined for self-braking. Experimental investigations of the racing regime were undertaken in order to verify the theoretical premises, three models of the axial hydroturbines being used for the purpose. Curves are furnished to show the distribution of velocities, pressures, energy along the radius, and the universal characteristics of racing are drawn up. An analysis of the data obtained enables the assertion to be made that the lowering of the number of revolutions in racing can be accomplished by raising the hydraulic resistances by artificial means in the part of the turbine between the inlet and outlet valves during racing or by reduction of the angle of the direction of the blades of the working wheel when this direction has no circulation. It is emphasized that the number of revolutions in racing measured on full-scale turbines should be greater than when measured on models. The distribution of hydraulic losses in the racing regime is discussed both as regards total losses and separately for the individual elements of the hydroturbine.

N. A. Kolokol'tsov

Courtesy Referativnyi Zhurnal, USSR

1555. Bannister, F. K., Induction ramming of small high-speed air compressor, *Proc. Instn. Mech. Engrs.* **173,** 13, 375–397, 1959.

Paper describes experimental and theoretical investigation of effect of inlet pipe on mass flow rate and required driving torque in high-speed single-cylinder air compressor. Experimentally measured effect of inlet pipe mass flow is increase (up to 18%) or decrease, depending on compressor speed and pipe length. Analysis considers finite-amplitude disturbances, but average wave propagation speed is taken everywhere equal to sound speed in undisturbed medium. An experimentally determined steady-flow characteristic is used to relate instantaneous pressure drop and mass flow rate through inlet valve in nonsteady operation, and cylinder pressure is assumed to be uniform at each instant. Analysis indicates fast approach to periodicity from initial quiescent state. Taking third cycle of analysis to represent periodic operation, theory predicts, for experimental conditions of maximum mass flow increment over the "no-pipe" condition, a mass flow increment of 16%, in good agreement with experiment. Effect of inlet pipe may be neutralized by closed-end side pipe branching from it close to the cylinder.

J. V. Foa, USA

1556. Wszelaczynski, A., Comparison with regard to constant pressure regulation efficiency of centrifugal compressors acting on the principle of choking in the suction duct, choking in the forcing duct or of adjustable diffusor vanes (in Polish), *Arch. Budown Maszyn* **6,** 3, 381–405, 1959.

Author analyzes and compares three methods of regulating the discharge pressure of a multistage compressor in terms of their effect on the over-all thermodynamic efficiency of the compressor. These methods consist of regulating the flow (1) in the discharge duct, (2) by means of adjustable vanes between compressor stages, (3) in the suction duct.

Using simplifying assumptions describing the performance and efficiency of a compressor stage at off-design conditions and postulating that all compressor stages have similar performance characteristics, author derives expressions for compressor efficiency for three cited methods of pressure regulation. He shows that, qualitatively at least, most efficient methods of maintaining constant pressure at compressor discharge could be aligned in the following order: method (3), method (2), method (1).

Author's qualitative conclusions certainly have validity within premises on which they were derived. This reviewer feels that the validity of conclusions is somewhat limited due to neglect of flow phenomena outside the compressor proper for the off-design conditions.

B. Zarwyn, USA

1557. Staritskii, V. G., An investigation of irregularity in the parameters of a flow in the vicinity of the controlling apparatus of an axial pump (in Russian), *Trudī Leningrad Politekhn. In-ta* no. 193, 51–59, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 4902.

Authors discovered during investigation of the structure of the flow in the region of the working parts of an axial pump that the divergence of balance in output in front of the working wheel amounted to no more than 1–2% at a time when measurements be-

tween the working wheel and the controlling apparatus and also beyond the controlling apparatus showed divergences exceeding 5%. The reason for divergences in output might have been the nonconformity between the measured and the mean peripheral magnitude of velocity because of the disruption of the circular symmetry of the field of velocities of the blade-lattice of the controlling apparatus. Special measurements were carried out for the field of velocities and the pressure in the vicinity of the controlling apparatus of the pump, with traversing of the flow by the radius and the spacing in order to investigate the disturbances introduced by the controlling apparatus, the object being to enable recommendations to be made regarding the selection of places for the measurements which would reduce the difference between the magnitude being measured and the mean peripheral to an insignificant figure. The following deductions resulted from the investigations made: (1) When measurements were made at points fixed in accordance with the surroundings the accuracy of the measurements proved to be insufficient. Appreciable extra precision is obtained by selecting points for measurement where the magnitudes are closest to the mean. In carrying out similar investigations for the pump it is essential to make measurements for spacing followed by averaging the results. (2) The irregularity of the field of velocities of the flow near the controlling apparatus may exceed 8%. This irregularity becomes larger with increase in curvature and relative thickness of the profile. (3) When changing the regime of the work of the pump by means of the revolution rate both the irregularity of the flow and the point on the periphery where the magnitude being measured is equal to the mean do not vary. When the regime of the work of the pump changes with output the irregularity of the flow increases with decrease of output, while the point where the magnitude being measured is equal to the mean alters its position. (4) The calculation of the pressure according to the mean value of the velocity gives a lower figure in comparison with the actual value of the full pressure. V. Kh. Abiants

Courtesy Referativnyi Zhurnal, USSR

1558. Niemand, L., The effect of friction upon gas expansion in the oblique cut of turbine vanes (in Polish), *Arch. Budown Maszyn* **6**, 3, 407–426, 1959.

Using basic equations of continuity, energy conservation and momentum conservation author obtains interesting results on the effect of friction on the main characteristics of the flow in the oblique cut of turbine vanes. Conclusions concerning the interdependence of the flow in the subcritical part and in the oblique cut, and on the limiting expansion possibilities are indicated.

W. Fiszdon, Poland

1559. Kolpakov, L. G., and Rakhimkulov, R. Sh., Designing and calculations for the hydraulic efficiency (k.p.d.) of profiles of the blades of turbo-drills (in Russian), *Sb. Tr. Ufimsk. Neft. In-ta* no. 2, 143–147, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 4873.

Results are given of the work done to replace the blades of a range of turbines of turbo-drills by blades the profiles of which were designed by M. I. Zhukovskii's method [Boiler-turbine construction, 1951, no. 5]. The method was developed graphically by L. A. Dorfman [Heat transfer and aerodynamics, Book 22, 1952]. A theoretical evaluation of the best of four constructed profiles demonstrated the possibility of increasing the efficiency (k.p.d.) of the turbine type T12M2-10" from 89 to 96.5%. The author states that the experimental results differ from the theoretical but he does not publish the data for the experiment.

N. A. Kolokol'tsov

Courtesy Referativnyi Zhurnal, USSR

1560. Iwasaki, M., Vortex theory of an airscrew, considering contraction or expansion of slipstream and variation of pitch of vortex sheets in it, *Rep. Res. Inst. Appl. Mech., Kyushu Univ.* **7**, 27, 159–232, 1959.

Work reported is a continuation of an earlier study [*Rep. Res. Inst. Appl. Mech., Kyushu Univ.* **6**, 23, 87–126, 1958]. Author proceeds to investigate mutual interference of airscrew vortex system and slipstream. Analytical results are reduced to tabular and graphical forms suitable for application in practical cases. Theory is then used to investigate some properties of a four-bladed windmill. R. A. A. Bryant, Australia

1561. Hodge, R. I., The cooling performance to two extruded-type air-cooled turbine nozzle blades, Aero. Res. Counc. Lond. Curr. Pap. 495, 43 pp., 1960.

Experimental study concerns temperature patterns and coolant flow losses for two full-sized gas-turbine nozzle blades tested in stationary cascade and cooled by air flow through sets of multiple internal passages. Blades had similar external profiles and were set at zero incidence angle. Ratio of coolant passage hydraulic diameter to blade chord was 8.3×10^{-3} for blade with profile reference number Z1881/13, and 5.8×10^{-3} for Z1881/18. Manifolding the coolant passage outlets was an advantage in blade temperature distribution. The 1881/18 blade had slightly lower surface temperatures and appreciably lower internal temperatures, but the coolant pressure drop was about five times as great. Surface lampblack flow patterns and temperature patterns are presented in detail. H. B. Nottage, USA

1562. Sergel', O. S., Semenov, I. A., and Lokotkov, Yu. E., Measurement of the pressure of injection of fuel into the pump-jet of a working motor (in Russian), *Trudi Mosk. Aviats. In-ta* no. 92, 103–128, 1957; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 4936.

It is difficult to obtain a diagram of the pressure of injection of fuel into a diesel pump-atomizer when the motor is operating, because of the inaccessibility of the strip of high pressure for fixing a gage transducer. Studies were undertaken for making and investigating quartz and strain-gage transducers for determining the pressure of the injection. The quartz transducer enabled measurements to be made of the force proportional to the pressure of the fuel in the force feed strip, while the strain-gage transducer measured the deformation of the lever of the gear of the pump-atomizer, depending on the pressure of the fuel on the plunger. The construction details of the quartz transducer were analyzed and the conditions operating on its static and dynamic calibration. It was shown that it was reliable in its working and that the assembly does not require alterations in the basic parts of the pump-atomizer. Experiments were carried out enabling the pressure to be measured (with an accuracy of 5 to 10%), and the continuity of spraying of the fuel in relation to various factors. The work of the strain-gage transducer was investigated in less detail. When tested it gave results close to those obtained when using the quartz transducer. V. M. Akimov

Courtesy Referativnyi Zhurnal, USSR

Flow and Flight Test Techniques and Measurements

(See also Revs. 1439, 1450, 1469, 1483, 1519, 1538, 1561, 1613, 1701)

1563. Collingbourne, J. R., and Pindar, A. C. S., Balance and pressure measurements at high subsonic speeds on a model of a swept-wing aircraft (Hawker P. 1052) and some comparisons with flight data, Aero. Res. Counc. Lond. Rep. Mem. 3165, 89 pp., 1960.

Basic results are given of measurements in the Royal Aircraft Establishment 10-ft × 7-ft subsonic wind tunnel over a range of lift coefficient at Mach numbers up to 0.93, the Reynolds number being 1.75 million. Though a really full analysis has not been possible, the data are discussed briefly and are compared with flight results on two aircraft of the type.

The tunnel-flight comparisons show reasonable agreement as regards the onset of longitudinal instability and the general characteristics of the pressure distributions on the wing. Full comparisons are not possible because of the absence of comprehensive and accurate flight measurements of pressure distribution.

From authors' summary

1564. Subramanian, N., and Rao, P. R., Measurement of viscosity of liquids by Poiseuille's method—a correction, *J. Sci. Indust. Res., India* **19** B, 9, 368–369 (Letters to the Editor), Sept. 1960.

A new correction factor, which takes into account the average excess pressure at the exit of a tube during the time of drop formation, has been suggested to be introduced in the Poiseuille equation for pressure (Hpg) in terms of the rate of flow (V). Experiments carried out with water and glycerine under low pressure heads to verify the modified equation show that the variation of H with V is linear under low rates of flow as predicted by the modified equation when the term (V^2/π^2a^4g) is negligibly small.

From authors' summary

1565. Rudinger, G., and Somers, L. M., Behaviour of small regions of different gases carried in accelerated gas flows, *J. Fluid Mech.* **7,** 2, 161–176, Feb. 1960.

The effect of acceleration in a gas flow on the motion of gas bubbles of different density is investigated experimentally and theoretically. Only one-dimensional flows accelerated by pressure wave are considered; the results apply, however, to accelerated flows caused by the curvature of streamlines. Authors note that the behavior of gas bubbles in accelerated flow is quite different from that of solid particles because a gas bubble cannot move relatively to the surrounding gas without being transformed in a vortex. The analysis considers first the initial acceleration of the bubble and its subsequent transformation in a vortex. The theory is based on the assumption that a spherical bubble is transformed in a vortex ring whereas a cylindrical bubble becomes a linear vortex pair. Experimental results with H_2, He, SF_6 bubbles in air, accelerated by shock wave of various strength, are given and agree well with the theoretical prediction.

The results are of importance in investigation where tracers are used for the visualization of streamlines or for the measurements of flow velocity. N. Zuber, USA

1566. Leaver, D. A. E., Visualization of secondary flow in the reservoirs of the slotted-wall working sections of the A. R. L. 12-inch and 30-inch water tunnels, Aero. Res. Counc. Lond. Curr. Pap. 483, 22 pp., 1960.

Author describes some flow visualization experiments in the Admiralty Research Laboratory water tunnels. By this technique he succeeded in minimizing secondary flows in the working section. The paper is of interest primarily to tunnel designers.

J. Laufer, USA

1567. Prior, B. J., and Hall, C. N., Subsonic wind-tunnel tests of various forms of air intake installed in a fighter-type aircraft, Aero. Res. Counc. Lond. Rep. Mem. 3134, 50 pp., 1960.

Tests have been made at Mach numbers up to 0.93 in the Royal Aircraft Establishment 10-ft × 7-ft high-speed wind tunnel to examine how the longitudinal characteristics of a fighter-type aircraft are affected by the installation of air intakes in the nose, fuselage sides, or wing roots. None of these intakes alters the lift or pitching-moment characteristics significantly, but their effects on external drag vary, and only the nose intake avoids an increase. This superiority derives principally from its low area ratio, the value of which largely determines not only the critical Mach number and hence the behavior at transonic speeds of the intake fairing but also its sensitivity to spillage and aircraft attitude.

Both forms of divided intake cause an increase in profile drag of roughly 20 per cent; on the side intake this is due to the locations of the boundary-layer by-passes and on the wing-root intake to the increased wing thickness. Their high area ratios make them sensitive to changes in flow direction and lead to high suction levels over the intake fairings which are expected to cause drag increases early in the transonic range. Separation in the canopy-intake junctions of the side intake reduces the drag divergence Mach number to 0.87, compared with 0.89 for the other models.

The shaping of the wing-body junction which was combined with the nose and side intake installations leads to a marked reduction in drag at transonic speed, an advantage not shared by the wing-root lay-out. From authors' summary

1568. Tirumalesa, D., Wall interference in transonic wind tunnels, *J. Aero. Soc. India* **11,** 3, 53–67, Aug. 1959.

The solution of the problem of wall interference in transonic wind tunnels in the two-dimensional and three-dimensional cases by means of the rheoelectric analogy is explained. This is applied for the evaluation of the wall interference in the two-dimensional case. The various types of walls considered are: (1) closed wall, (2) open jet, (3) perforated wall, (4) slotted wall. In each case, the rheoelectrical representation of the boundary conditions is explained in detail for the total velocity potential in the tunnel as well as for the wall perturbation potential. Then author describes the various experiments conducted for different values of the ratio (b/l) ($2l$–chord of the profile, $2b$–height of the tunnel). The results are divided into two parts: (a) the thickness effect or the symmetrical problem, (b) the lifting problem or the incidence effect. The free air solution is taken as that due to a uniform distribution of sources and sinks (vortices) along the chord of the profile when one considers the thickness effect (incidence effect). The correction velocities along the chord of the profile are presented for the various configurations considered. These results are discussed and compared with those obtained by others where the free air solution was taken as that due to a single doublet (vortex). From author's summary

1569. McCloud, J. L., III, Hall, L. P., and Brady, J. A., Full-scale wind-tunnel tests of blowing boundary-layer control applied to a helicopter rotor, NASA TN D-335, 34 pp., Sept. 1960.

A full-scale wind-tunnel test was conducted of two boundary-layer-control applications to a 44-ft diameter helicopter rotor. Blowing from a nozzle near the leading edge of the blades delayed retreating blade stall. Results also indicated that delay of retreating blade stall could be obtained by cyclic blowing with a lower flow rate than that required for continuous blowing.

It was found that blowing applied through a nozzle at mid-chord had no effect on retreating blade stall.

From authors' summary

1570. Gregory, N., and Walker, W. S., Wind-tunnel tests on the use of distributed suction for maintaining laminar flow on a body of revolution, Aero. Res. Counc. Lond. Rep. Mem. 3145, 32 pp., 1960.

Experiments were carried out in the National Physical Laboratory 13-ft × 9-ft wind tunnel on a 15-ft long body of revolution of fineness ratio 10 : 1. Observations were made of the effects of Reynolds number, yaw, and of isolated excrescences on the position of transition on the solid body, and the possibilities of in-

creasing the extent of laminar flow by means of area suction were examined.

At zero angle of yaw, the maximum Reynolds number at transition on a solid version of the body was 4½ million. When the body was set at a small angle of yaw, the transition position was much farther forward along certain generators of the body than on others, owing to the instability of the three-dimensional boundary layer. The critical heights of small conical excrescences which just precipitated transition were found to be much the same as those required on a two-dimensional airfoil.

The "suction" version of the body was porous over the central third of the length and was attached to the overhead tunnel balance by means of a porous wing. There appeared to be no fundamental difficulty in obtaining extended laminar flow over the body with distributed suction up to a tunnel speed of 80 ft/sec. Above this speed, the skin joints were too rough, and numerous wedges of turbulence originating from the joints spoiled the laminar flow. The adverse effects of yaw which were noticed on the solid body were not encountered and far-back transition was obtained with the same suction quantity as was needed at zero yaw. This suction quantity was large ($v_0/U_0 \simeq 0.0009$ on the body), because of the rough surface.

The suction applied at the wing-body intersection (locally $v_0/U_0 \triangleq 0.003$) reduced the secondary flow in the boundary layer and enabled laminar flow to be maintained in this region at a wind speed of 80 ft/sec despite experimental difficulties over awkward skin joints. Even with laminar flow right round the intersection to the trailing edge of the wing, the laminar wake from the wing rapidly became turbulent downstream of the trailing edge and gave rise to a spreading wedge of turbulent boundary layer on the body. Further investigation of this difficulty would best be carried out on a simple part model, such as the junction of a stub wing with a flat plate.

Partly in consequence of the large suction quantity needed for laminar flow, and partly because the flow on the body in the wake of the wing and aft of 0.82 of the body length remained turbulent when suction was applied, there was only a small reduction in the effective drag coefficient of the model with suction. The analysis shows, however, that the potential gains at high Reynolds numbers would be large, provided the porous surface were satisfactorily smooth and extended sufficiently far back to enable full-length laminar flow to be achieved. From authors' summary

1571. Sivo, J. N., Meyer, C. L., and Peters, D. J., Experimental evaluation of rocket exhaust diffusers for altitude simulation, NASA TN D-298, 43 pp., July 1960.

Investigation to evaluate methods of minimizing over-all pressure ratio required to establish and maintain full expansion of the nozzle flow. Configurations investigated were cylindrical diffusers, diffusers with contraction, and diffusers including a right-angle turn.

Minimum length-diameter ratio of cylindrical diffusers was about 6 for minimum pressure-ratio requirements. With cylindrical diffusers of adequate length, pressure-ratio requirements were primarily a function of the ratio of diffuser to nozzle-throat areas and were essentially independent of primary-nozzle type (including two clustered configurations) or area ratio. Two check points obtained with rocket engines indicated the pressure-ratio requirements at given ratios of diffuser to nozzle-throat areas were lowered, as compared with the requirements with air, as a result of the reduced ratio of specific heats.

Minimum length-diameter ratio of the contraction throat of convergent-divergent diffusers was also about 6 for minimum pressure-ratio requirements. With adequate contraction-throat length, the pressure-ratio requirements of such diffusers were appreciably below those of comparable cylindrical diffusers when used with conical and cutoff-isentropic nozzles, but not when used with a bell nozzle.

Minimum pressure-ratio requirements of a diffuser including a simple long-radius right-angle turn at maximum diffuser area, obtained with the center of radius of the turn a minimum of 2 diffuser diameters downstream of the nozzle exit, were not appreciably above those of a comparable optimum cylindrical diffuser. Diffuser including a long-radius right-angle turn at a contraction minimum area had somewhat lower pressure-ratio requirements than the aforementioned simple turn. W. F. Davis, USA

1572. Tinajero, A. A., and Fresh, J. N., Aerodynamic response of a 7-foot ground-effect machine flying over uneven surfaces, David W. Taylor Mod. Basin Rep. 1436 (Aero. Rep. 982), 30 pp., June 1960.

The results of dynamic and hovering tests of a 7-ft ground-effect machine flying over uneven surfaces is presented. The aerodynamic response when cruising over uneven surfaces is evaluated and compared to results obtained from the hovering stability parameter in pitch. It was found that ground-effect machines having a peripheral nozzle, when flying over surfaces with a sine-wave contour and increasing forward velocity, will eventually encounter a critical speed at which the pitch of the vehicle will be out of phase with the surface. If the amplitude of pitch becomes too great, collision will occur. The critical speed depends on the height at which the machine is flying, the ratio of vehicle length to surface wavelength, and the amplitude of surface wave. In general, it can be said that the critical speed is a function of the stability derivative in pitch, the mass moment of inertia of the machine, the height at which the vehicle is flying, and the local acceleration of gravity. From authors' summary

1573. Rasmussen, M. L., Determination of nonlinear pitching-moment characteristics of axially symmetric models from free-flight data, NASA TN D-144, 51 pp., Feb. 1960.

Author considers a missile whose yawing moment may be expressed as the sum of two terms which are linear and cubic, respectively, in the angle of attack. For sufficiently small constant rates of roll, and zero damping, the nonlinear equations of motion are solved in closed form in terms of elliptic integrals of the first and third kind (whereas circular functions arise in linearized theory).

The frequency of the resulting motion is shown to be approximately linear in the squares of the minimum and maximum amplitudes of this motion with coefficients which depend upon those defining the yawing moment. The precession of a spinning missile also depends on these quantities.

A technique for estimating the coefficients of the two terms in the restoring moment from two sets of test data is derived, and it is shown how the effect of damping may be incorporated.

S. Kirkby, England

1574. Ficken, N. L., and Lo, H. F., Study of stack-gas flow on LSD28- and LSD32-class ships from tests with model 4552 in the circulating-water channel, David W. Taylor Mod. Basin Rep. 1335, 23 pp., July 1960.

A test technique was developed by which stack-gas flow patterns can be predicted from tests in the circulating-water channel. Tests were conducted with a model of the superstructure of LSD28. It was determined that increased stack height and/or increased gas velocity is required to eject the gas beyond the turbulent boundary zone. Tests were conducted also with models of the superstructures of LSD28 and LSD32-35 fitted with TMB-designed stacks. The flow was observed to be acceptable with gas velocities in excess of twice the relative wind velocity.

From authors' summary

Thermodynamics

(See also Revs. 1216, 1474, 1481, 1504, 1524, 1545, 1556, 1564, 1638, 1690)

Book—1575. Bird, R. B., Stewart, W. E., and Lightfoot, E. N., Transport phenomena, New York, John Wiley & Sons, Inc., 1960, xxi + 780 pp. $13.75.

This book provides a new unified approach to the theory and analysis of processes in which momentum, heat, or matter is transported by conduction or diffusion, or by turbulence. In effect, intermediate-level texts on fluid dynamics, heat transfer, and diffusion have been coordinated and combined, yielding a general text on transport theory specially suitable for first-year graduate students in chemical engineering, but also of interest to many mechanical and other engineers. Topics covered are explained and interpreted for ready comprehension. Discussion questions and problems with and without answers follow each chapter.

The momentum transport chapters cover viscosity and its interpretation and prediction, laminar and turbulent flow velocity distributions, interphase transport (friction), and the differential and macroscopic equations of change. In another section of equal length (240 pages), thermal conductivity and conduction, temperature distributions in solids and in laminar and turbulent flow, interphase transport (heat-transfer coefficients), and the differential and macroscopic heat balances are considered. The same general topics are then covered again, similarly, for diffusion and mass transport. An appendix summarizes vector and tensor notation.

Besides the homogeneity of treatment above apparent, many tables list equivalent or analogous forms of similar equations, for ready comparison, and worked examples are frequent. The treatment is definitely rigorously mathematical, but not at an exaggerated level for its utilization. C. F. Bonilla, USA

1576. Zettner, T., Variable relations of steam turbines and condensers (in Russian), *Energia es Atomtechnika* **12,** 2/3, 145–158, Feb./Mar. 1959.

1577. Chambadal, P., Mixing gases and Gibbs' paradox, *Engineering* **190,** 4926, 398–399, Sept. 1960.

Gibbs' paradox presents a discontinuity in the usual theory of gas mixtures which cannot easily be justified. A new solution is presented. From author's summary

1578. Ziebland, H., and Burton, J. T. A., The thermal conductivity of heavy water between 75° and 260°C at pressures up to 300 atm, *Inter. J. Heat Mass Transfer* **1,** 2/3, 242–254, Aug. 1960.

The thermal conductivity of heavy water having an isotopic purity of 99.85% was measured with a vertical coaxial cylinder apparatus at pressures between 24 and 294 atm in the temperature range 75° to 260°C. The pressure range of the experiments included the critical pressure (218 atm).

Identical with the trend exhibited by water, the thermal conductivity of heavy water was found to increase with rising temperature to a shallow maximum near 110°C and to fall with a further rise in temperature. The influence of pressure on thermal conductivity was small, and at 75°C amounted to an increase of 0.77% for a pressure rise from 1 to 100 atm. This is similar to the values reported for natural water by previous investigators.

In comparison with natural water, the thermal conductivity of heavy water, in the range investigated, was always lower. According to these tests the ratio k_{H_2O}/k_{D_2O} increased almost linearly from 1.045 at 75°C to a value of 1.134 at 260°C.

No measurements of the thermal conductivity of heavy water have previously been reported at temperatures above 82°C. Below this temperature the present data are in good agreement with earlier work. From authors' summary

1579. Sherman, A., and Martinek, F., Thermodynamic and electrical properties of mercury vapor at pressures below atmospheric and high temperatures, AFOSR TN 60-657 (General Electric Co., Flight Propulsion Lab. Dept.), 39 pp., Feb. 1959.

The thermodynamic properties of mercury vapor have been calculated for temperatures up to 15,000°K and pressures between 1 and 10^{-4} atmospheres. The first ionization is shown to be essentially complete at 15,000°K. At the lower pressures the ionization is completed earlier; however, only first-degree ionization was considered. Corrections have been included to account for the lowering ionization potential and the finite limit of the electronic term series. The largest corrections occur at the higher pressures, the correction at 10^{-4} atmospheres being negligible. For example, the temperature required to give 95% degree of ionization at one atmosphere is corrected by approximately 2%.

The electrical conductivity is calculated in the absence of a magnetic field as well as in the presence of one. The corrections due to a 10,000 gauss field, assuming Hall currents are free to flow, are found to be very important at low pressures. For example, at 10^{-4} atmospheres and 15,000°K the conductivity has been lowered by six orders of magnitude. In addition, the conductivity, which previously rose monotonically, can now have a maximum at the lower pressures. From authors' summary

1580. Bailey, J. A., Design pressure for a reactor containment vessel, *Proc. Amer. Soc. Civ. Engrs.* **86,** ST 2(*J. Struct. Div.*), 23–32, Feb. 1960.

The method of calculation is based on a first law (of thermodynamics) analysis of the condition in a reactor containment vessel at the rupture in the primary fluid system. Equilibrium of liquid, vapor, and air was assumed and the dynamic effects are neglected. This method gives the minimum pressure in the containment vessel while design efforts would need the maximum pressure. A pertinent treatment should include, at least, evaporation rate (a basic reference could be: J. Frenkel "Kinetic theory of liquids," Dover, 1955) and a wave phenomena (a spherical wave would give an adequate approximation); and preferably reaction rate (such as absolute reaction-rate concept, H. Eyring, V. M. Polanyi, *Z. Physik Chem.* (*B*)**12,** p. 279, 1931). It should be pointed out that even if only equilibrium analysis is required, air above 300 psia deviates significantly from perfect gas law, especially at such moderate temperatures. Enthalpy-entropy diagram of air is included in many elementary texts [such as Mooney, "Thermodynamics," Prentice-Hall, 1953; AMR **7**(1954), Rev. 285]. S. L. Soo, USA

1581. Pavlov, I. I., Investigation of the structure of the flow of a two-phase mixture in a pipe (in Russian), *Nauchn. Trudi Mosk. Lesotekhn. In-ta* no. 9, 189–202, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5300.

Experiments are described of investigations on the structure of an air-water flow in a vertical plexiglas tube with a length of 9.50 m and a diameter of 25.48 mm. In the course of the tests moving pictures were taken in four sections at various heights in the tube with different correlations between the inputs of the water and the air for a number of values of the input of water when using different constructional types of mixer; the relation of the consumption of air to the comsumption of water, representing the basic parameter in the motion of the gas-liquid mixture, did not exceed nine. The working-over of the test data was undertaken to obtain empirical relations from the above-mentioned parameters for some second-degree geometrical and kinematic characteristics of the flow (the dimensions and speed of mixing of the air locks when the mixture had a beady structure, and the distance from the mixer to the section in which observations were being made on the transition from an emulsion to a beaded structure).

V. A. Arkhangel'skii
Courtesy Referativnyi Zhurnal, USSR

Book—1582. Callen, H. B., Thermodynamics: An introduction to the physical theories of equilibrium thermostatics and irreversible thermodynamics, New York, John Wiley & Sons, Inc., 1960, xv + 376 pp. $8.75.

The author's aim is to abandon the traditional inductive approach to the presentation of phenomenological thermodynamics, and to develop this branch of science from a number of basic postulates, similar to the theories of analytical mechanics and electromagnetism. Such an objective is extremely worthwhile, as it obviates certain irrelevant manipulations, frequently employed in the derivation of thermodynamic laws. However, after seeing how Dr. Callen has gone about attaining his objective, this reviewer is unable to share the enthusiasm aroused in other readers by this text.

Since this book is primarily intended as a college textbook, a review must necessarily keep this fact in mind. It is customary to build the structure of thermodynamics in steps, introducing the four laws of thermodynamics (including the "zeroth law") one by one, thus clarifying the contribution which each law makes to the whole. After each step, a new class of physical phenomena becomes accessible to analysis. By introducing the Second Law in Carathéodory's form, its unique position among the laws of nature can be pointed out. Now, it is possible to replace each of the four laws by an equivalent postulate (to which must be added two laws from classical mechanics), and again to construct thermodynamics step by step. There is a great pedagogical advantage in this procedure.

Dr. Callen does not avail himself of this advantage; his break with tradition is so complete that the "Laws of Thermodynamics" are not even mentioned by name. The postulates upon which he builds his structure of thermodynamics are all put at the beginning, some openly labeled as postulates and some hidden, as e.g. the statements on p. 16 "there exist walls..." and on p. 17 "heat flux to a system...," or the equations (2.3), (2.4), (2.5) which introduce temperature, external force, and chemical potential. The author did not attempt to reduce the number of required postulates to a minimum (preface), and his subsequent development is concentrated on those phenomena where all thermodynamic postulates are involved. However, a great number of problems involving energy conversions can be analyzed without the concepts of entropy or temperature, or both. Such problems are not discussed at all in this text. All this is, in reviewer's opinion, most unfortunate.

The subject matter discussed in the preceding paragraph constitutes the first chapter of Dr. Callen's book. Even those who do not subscribe to his presentation will find many valuable statements in this chapter. Beginning with Chapter 2, Dr. Callen's text is not basically different from other texts after the introduction of the second law of thermodynamics. However, each chapter bears the mark of its author's individuality, and each contains many topics not found in other thermodynamics books (to the exclusion of others, to be sure, which are usually discussed).

Chapters 2 to 11 deal with general principles: conditions of equilibrium (confusing only the use of d for virtual changes), differential relationships, a somewhat abstract discussion of reversible vs. irreversible processes, the T^* method for the analysis of processes near zero temperature, a discussion of Legendre transformations and their use in the construction of new thermodynamic functions, criteria of stability, Le Chatelier-Braun principle, phase transitions, Tisza's theory of second-order phase transitions, phase diagrams; Nernst's theorem, and extension of the laws for simple systems to those for general systems subjected to forces other than an external pressure.

Such systems are discussed in Chapters 12 to 14: systems undergoing chemical reactions (gas phase only), solid systems, and magnetic and electric systems. The last two of these chapters are a principal asset of this book.

The last three chapters (15 to 17) deal with fluctations and irreversible thermodynamics. The chapter on fluctuation is a very convenient resume of some statistical concepts, which form the transition from phenomenological to statistical thermodynamics.

Chapter 16 on irreversible thermodynamics is a disappointment. This ten-page chapter does not do justice to the practical or the pedagogical value of the subject. One had hoped for a more elaborate treatment from such an authoritative writer; certainly, a great opportunity has been wasted. This chapter does not even contain a survey of the principal theorems of irreversible thermodynamics.

Chapter 17 is a fine presentation of irreversible coupling processes involving coupling of heat current with electrical current, but no other irreversible processes are discussed.

In summarizing the appraisal of this book it may be stated that, due to its concentration on concepts, this book does not fulfill the needs of someone looking for information on specialized applications of thermodynamics; it is, however, a valuable addition to the library of anyone working in theoretical thermodynamics; this reviewer has already repeatedly found use for it. As a textbook, Chapters 2 to 15 are excellent, but additional material should be employed to supplant the axiomatics of thermodynamics and to supplement the material in Chapter 16.

E. F. Lype, USA

1583. Fastov, N. S., The thermodynamics of irreversible processes in plastic deformation, *Soviet Phys.-Doklady* **5,** 1, 160-163, July/Aug. 1960. (Translation of *Doklady Akad. Nauk SSSR (N.S.)* **130,** 3, 541-544, Jan./Feb. 1960 by Amer. Inst. Phys., Inc., New York, N.Y.)

In the theory of relaxation processes, it is tacitly assumed that the relaxation-time spectrum of the body remains constant during the deformation, and this is equivalent to neglecting the variation of the macrostructure of the body and, in particular, the plastic deformation. In plastic deformation, the relaxation spectrum corresponding to an increasing load differs from that corresponding to a decreasing load, as is shown by the change in macrostructure during the deformation.

Author considers small elastic-plastic deformation. In the deformation process, both the relative volume ε_{ee}, and the pure shear deformation $\varepsilon_{ik} - {}^1/_3\, \varepsilon_{ee}\, \delta_{ik}$ usually vary. However, in view of the fact that the volume of the body does not vary during the plastic deformation occurring after the removal of the load, author limits his investigation of the general properties of plastic deformation to the case of pure shear. From author's summary

1584. Hollander, E. F., The application of the thermodynamics of irreversible processes to welding arcs (in English), *Czech. J. Phys.* **9,** 2, 229-236, 1959.

The author presents an empirical relation between voltage-current characteristics of the power source and the voltage-current characteristics of a welding arc including the electrode melt rate. After linearizing this relation he is able to discuss the stability of the length of arc. He shows that his relation is not inconsistent with the thermodynamics of irreversible processes.

His results indicate that the dynamic resistance of the source should be just a little less than the dynamic resistance of the arc. This is compatible with the operation of a "hissing" arc but, in the reviewer's opinion, shows clearly the restrictive effect of his linearization upon the general stability discussion.

E. E. Covert, USA

1585. Konowalow, D. D., and Hirschfelder, J. O., Intermolecular potential functions for nonpolar molecules, Univ. Wisconsin, Theoretical Chem. Lab., WIS-AF-17, Series 5, 34 pp., Oct. 1960.

The Morse potential function is used to represent the intermolecular potential for several nonpolar substances. The potential con-

stants are determined from a combination of crystal structure and second virial coefficient data for Ne, Ar, Kr, Xe, CH_4, and N_2. Over a wide temperature range the theoretical second virial coefficients determined from the Morse potential for these substances agree very well with experimental data and are quite comparable with calculations using the Buckingham (exp-6) or the Lennard-Jones (12-6) potentials. For Kr, it is found that the agreement with the experimental second virial coefficients is greatly improved by dividing the intermolecular potential into two separate Morse functions, one applicable where the potential is negative and the other applicable where the potential is positive.

From authors' summary

1586. Lewis, R. M., Measure—theoretic foundations of statistical mechanics (in English), *Arch. Rational Mech. Anal.* **5**, 5, 355–381, July 1960.

The basic formulas of classical equilibrium statistical mechanics are derived from well-known theorems in measure theory and ergodic theory. The method used is a generalization of the methods of Khinchin and Grad and deals with several, in fact a "complete set" of, "invariants" or "integrals of the motion." Most of the results are simple corollaries of Birkhoff's ergodic theorem, and since time-averages are used, the whole approach is characterized by an absence of statistical "ensembles" and probability notions. In the course of the development a "generalized temperature" is introduced, and a generalization of the second law of thermodynamics is derived. Formulas for the "microcanonical", "canonical", and "grand canonical" distributions appear as special cases of the general theory.

From author's summary

1587. Lebowitz, J. L., Frisch, H. L., and Helfand, E., Non-equilibrium distribution functions in a fluid, *Physics of Fluids* **3**, 3, 325–338, May/June 1960.

The one- and two-particle distribution functions f_1 and f_2 in a liquid are determined as solutions of the Fokker-Planck equations of Kirkwood, to terms linear in deviations from local equilibrium. The methods of Chapman, Enskog and Grad are used, but a justification is not given for the omission of all terms $\alpha^{(n_1,n_2)}\chi(\nu_1)\chi(\nu_2)$ in the series expansion (4.21) for f_2 in Grad's method.

It is shown that f_1 and f_2 maximize the entropy, subject to the constraints of given densities and fluxes. Alternatively, they maximize also the entropy plus entropy productions in appropriate characteristic times.

P. E. Kriezis, Greece

1588. Vineyard, G. H., Molecular distribution functions involving two times, *Physics of Fluids* **3**, 3, 339–345, May/June 1960.

Paper develops the kinetic theory of distribution functions giving the probability that certain molecules of a fluid have given positions, velocities and accelerations at two different times. These probabilities determine time-displaced correlation functions, such as the time-displaced radial density function.

P. E. Kriezis, Greece

Heat and Mass Transfer

(See also Revs. 1184, 1185, 1445, 1468, 1474, 1478, 1482, 1490, 1499, 1505, 1531, 1561, 1575, 1578, 1584, 1587, 1588, 1628, 1629, 1643)

Book—1589. Veinik, A. I., Approximate calculation of heat conduction processes [Priblishennii raschet protsessov teploprovodnosti], Moscow, Gosudarstvennoe Energeticheskoe Izdatel'stvo, 1959, 183 pp. + 96 figures.

Book is an exposition of author's simplified method of solving heat-conduction problems. Consider a slab $2X_0$ wide of original temperature zero. Apply at time $t = 0$ the surface temperature 1. Then one may visualize the temperature distribution in the half slab as given by

$$T = (1 - x/X)^n$$

where x is the inward distance from the surface, X the thickness of the thermal boundary layer. The growth of X with t is determined from the differential equation

$$\frac{\gamma c}{n+1} dX = \frac{nk}{X} dt$$

which is obtained by placing the inflowing heat, $dQ = -k[\partial T/\partial x]_0 dt$ equal to the absorbed heat $dQ = \gamma c \frac{d}{dX}\left[\int_0^X T dx\right] dX$. One finds that X grows as the square root of time. The exponent n is determined by comparing the approximate solution with the known rigorous (error function) solution of the problem. One finds that the best value of n is somewhat less than 2. The use of $n = 2$ is recommended. The foregoing approach is appropriate until X reaches the magnitude X_0. Thereafter the temperature distribution in the slab may be described by

$$T = T_c + (1 - T_c)(z/X_0)^m$$

where T_c is the midplane temperature, and z is the outward distance from the midplane. Again a heat flow balance is taken, and that establishes a functional relation between T_c and t. By comparison with the rigorous solution the best exponent appears to be about $m = 1.5$.

Author determines the X and the T_c variations with time, and proposes exponents n, m, for slabs, hollow cylinders (spheres) heated from within, solid cylinders (spheres) heated from without, when the surface condition is (a) isothermal, (b) constant heat flux is prescribed, (c) surface Biot number is prescribed, (d) contact is established between two solid bodies, (e) a freezing phase adjoins a liquid phase. (The sphere problem is omitted in the latter case.) The book concludes with a very brief section on complex bodies, utilizing the notion of "reduced radius," and with 6 pages of tables of material properties. The latter contains, for instance, the information that the viscosity of liquid copper at 1100°C is 0.00037 kgweight sec/m^2, which corresponds to 0.036 poises. (This is a piece of information which the reviewer has been unable to locate anywhere else.) The boundary-layer method for the early phases of heat conduction, though motivated somewhat differently, has been extensively used also in this country (see e.g. T. R. Goodman, AMR **11** (1958); Rev. 651); the characterization of the later phase appears novel to this reader.

Reviewer has one minor complaint: the book gives too much detail in the repetition of similar derivations.

G. Horvay, USA

1590. Kogan, M. N., On flows with large heat conductions, *Soviet Phys.-Doklady* **4**, 5, 974–976, Mar./Apr. 1960. (Translation of *Doklady Akad. Nauk SSSR (N.S.)* **128**, 3, 488–490, Sept./Oct. 1959 by Amer. Inst. Phys., Inc., New York, N. Y.)

Basic assumption is that the viscous boundary layer is small compared with the thermal boundary layer. Flow and temperature in the thermal boundary layer are found in the form of similarity solutions for arbitrary mainstream velocity and temperature distributions as well as arbitrary wall temperature distribution. Heat transfer is calculated. Flows with very large heat conductivities are considered qualitatively.

D. C. Leigh, USA

1591. Stonecypher, T. E., Periodic temperature distribution in a two-layer composite slab, *J. Aero/Space Sci.* **27,** 2, 152–153 (Readers' Forum), Feb. 1960.

1592. Hatton, A. P., Thermal conductivity and diffusivity measurements by an unsteady-state method with application to insulating materials containing moisture and ice, *J. Mech. Engng. Sci., Lond.* **2,** 1, 45–51, Mar. 1960.

Simultaneous measurements of thermal conductivity and thermal diffusivity of porous insulating materials are described. The method employs a finite slab with a known heat flux applied at time $t = 0$ to one surface. The other surface is maintained at constant temperature and the temperature at an intermediate position is measured at subsequent times. Tests carried out by the author show the method to be surprisingly insensitive (in this reviewer's opinion) to errors due to three-dimensional effects, contact resistance, etc. An appendix by G. Roberts describes an open-loop amplifier circuit used to provide a null deflection method of measurement of the continuously varying temperature.

J. R. Moszynski, USA

1593. Lotkin, M., The calculation of heat flow in melting solids, *Quart. Appl. Math.* **18,** 1, 79–85, Apr. 1960.

Author treats mathematically a vital engineering problem. With minor modifications, analysis is applicable to most ablation reentry problems. A numerical solution of differential equations for a finite slab conducting heat and with a receding boundary is given. Techniques may be used for thermal protection design of missiles. Reviewer used these techniques with satisfactory results. Comments in AMR **12**(1959), Rev. 3071 are applicable.

H. Hurwicz, USA

1594. Edamoto, I., Kanayama, M., Munakata, S., and Honda, T., On the cooling of an ingot and the mould during casting: Part 1, The case when two parallel infinite metal plates are used as the mould; Part 2, The case when a hollow metal cylinder of infinite length is used as the mould, *Technol. Rep. Toboku Univ.* **24,** 2, 1–24, 1960.

Part I. Neglecting the heat developed during solidification, authors determine the effect of contact resistance upon rate of cooling of slab of molten metal of half-width a in a container of width $b - a$. They solve the heat-conduction equation in the two media, assuming temperature expression of type $\theta = \int A(\lambda)\cos\lambda\chi e^{-\lambda^2 a^2 t} d\lambda$ in the melt, and a similar expression in the container. The above expression satisfies some of the boundary conditions for arbitrary $A(\lambda)$. The function $A(\lambda)$ and the integration path are then so determined that all initial and boundary conditions be satisfied.

In Part II the same problem is solved for cylindrical symmetry, and the results compared with the well-known experimental data of Pellini and associates. (Reviewer's note: The much simpler problem, $b - a = \infty$, was solved by reviewer and R. H. Edsall, *Trans. AIME,* **218,** p. 927, 1960, by a simple Laplace transform, and likewise compared with the Pellini experiments.)

G. Horvay, USA

1595. Lu, P.-C., Combined free and forced-convection heat-generating laminar flow inside vertical pipes with circular sector cross sections, *ASME Trans.* **82** C (*J. Heat Transfer*), 3, 227–232, Aug. 1960.

A theoretical analysis of combined free and forced convection, heat-generating laminar flow inside cylindrical pipes. The coordinate system was established so that the elements are parallel to the direction of the generating body forces, such as gravity or centrifugal force. A numerical example is presented to illustrate the use of the mathematical relations developed.

G. A. Hawkins, USA

1596. Hall, J. G., and Golian, T. C., Heat transfer to sharp and blunt yawed plates in hypersonic airflow, AFOSR TN 60-938 (Cornell Univ., Cornell Aero. Lab., Inc. Rep. AD-1052-A-11), 8 pp., Sept. 1960.

1597. Petrovichev, V. I., Heat transfer to mercury in a circular tube and annular channels with sinusoidal heat load distribution, *Inter. J. Heat Mass Transfer* **1,** 2/3, 115–120, Aug. 1960.

From experiments on heat transfer to liquid mercury in turbulent flow with sinusoidal heat input distribution along the channel length, an equation is developed relating the maximum wall temperature to the Peclet number. Data are presented for both circular-tube and annular channel configurations of length-to-diameter ratios from 44.6 to 170. Experimental values of the local Nusselt number are found to be in fair agreement with those predicted by a turbulent thermal diffusivity approximation.

Reviewer found paper difficult to read because nomenclature is not well defined. For example, expression for maximum wall temperature includes two temperatures which are not defined, and the temperature at which the Peclet number is evaluated is not specified. Reviewer notes that most papers in this valuable new Journal contain a table of nomenclature and suggests that the editors insist on such a table for all papers.

R. L. Young, USA

1598. Ermolin, V. K., Local and average heat transfer coefficients at an air stream in a tube with a pointed inlet, *Inter. J. Heat Mass Transfer* **1,** 2/3, 147–151, Aug. 1960.

Tests were made to determine influence of length-to-diameter ratio for transfer of heat from air to colder wall. Tests were conducted at atmospheric pressure, air entering tube in range 340°C to 421°C, then cooled by water boiling in individual pockets along outside length of tube. Author recommends as working equation for fluids in turbulent range: $Nu = 0.023\, Re_f^{0.8} Pr_f^{0.4}(1 + k(D/L))$ where k is correction factor, depending on Reynolds number, being about 2 at $Re = 20{,}000$ and increasing to 5 at $Re = 4000$. Above L/D (length-to-diameter ratio) of 30, value of k decreases, approaching zero above L/D of 40.

Tests were run with air inside horizontal cylindrical steel tube of 31.66-mm diameter × 1236-mm length. Reviewer finds results in agreement with prior work of others. Since tests were made only with air, reviewer considers relationship is limited to fluids with comparable Prandtl number.

D. Aronson, USA

1599. Sparrow, E. M., and Siegel, R., Application of variational methods to the thermal entrance region of ducts, *Inter. J. Heat Mass Transfer* **1,** 2/3, 161–172, Aug. 1960.

Authors apply Ritz variational methods for approximate determination of the lower eigenvalues and eigenfunctions associated with the analysis of convective laminar heat transfer in the thermal region of ducts. Uniform wall heat flux is assumed. In the assumed forms for the solution, orthogonality conditions, in addition to the boundary conditions, are required to be satisfied, and the remaining coefficients (within a constant factor) and the eigenvalues are then determined from the variational condition. In the case of a circular tube and a parallel-plate channel, problem involves Sturm-Liouville type of ordinary differential equations, and the variational method applied here is found to give very good agreement with known exact solutions for the second eigenvalues β_2^2 and eigenfunctions R_2. ($\beta_1^2 = 0$, $R_1 = $ const. in all cases here.) In case of square duct, eigenvalue problem involves a partial differential equation, and authors again derive approximate β_2^2 and R_2 by the variational method. In this latter case, comparison is also made with results obtained by applying an idea of Millsaps and Pohlhausen. Paper is very clearly written.

Misprint: In Eq. (4a), "β_n" should be replaced by "β_n^2."

M. Morduchow, USA

1600. Abbrecht, P. H., and Churchill, S. W., The thermal entrance region in fully developed turbulent flow, *AIChE J.* **6,** 2, 268–273, June 1960.

The temperature profile and the local rate of heat transfer from the wall were measured at 0.453, 1.13, 4.12, and 9.97 tube diameters downstream from a step increase in wall temperature for air in fully developed turbulent flow at Reynolds numbers of 15,000 and 65,000 in a 1.52-in. tube. The velocity profile and the pressure were also measured at these lengths.

Radial and longitudinal temperature gradients, radial heat fluxes, and eddy diffusivities for heat and momentum transfer were computed from the measurements. The longitudinal temperature gradients at all radii were found to differ significantly from the mixed mean temperature gradient. Although the radial heat flux was a maximum at the wall, the radial heat-flux density, in terms of which the eddy diffusivity for heat transfer is usually defined, was found to go through a maximum near the wall and then to decrease almost linearly across the thermal boundary layer. The eddy diffusivity for heat transfer was found to be independent of length in the thermal entrance region and hence a function only of the fluid motion, as previously hypothesized.

From authors' summary by L. M. Grossman, USA

1601. Eckert, E. R. G., and Irvine, T. F., Jr., Pressure drop and heat transfer in a duct with triangular cross section, *ASME Trans.* **82** C (*J. Heat Transfer*), 2, 125–138, May 1960.

Friction factors have been measured for a duct whose cross section has the shape of an isosceles triangle with a side ratio 5 to 1 in the fully developed flow region for laminar, transitional, and turbulent conditions. In addition, local and average heat-transfer coefficients and the temperature field in the duct wall have been determined for the condition of constant heat generation per unit volume of the duct walls.

Friction factors in laminar flow agreed well with analytical predictions. In the turbulent flow range they were lower by 20% than values calculated from relations for a round tube with the use of the "hydraulic diameter." Heat-transfer coefficients averaged over the circumference of the duct were only half as large as values calculated from round tube relations in the Reynolds number range from 4300 to 24,000. The measurements also revealed that thermal starting lengths were in excess of 100 diameters. In round tubes a length of 10 to 20 diameters has been found sufficient to develop the temperature field. [From authors' summary]

Reviewer's note: The tests have been performed with air and, therefore, relate only to a Prandtl number of about 0.7.

H. Blok, Holland

1602. Baxter, C. B., and Davies, D. R., Heat transfer by laminar flow from a rotating spherical cap at large Prandtl numbers, *Quart. J. Mech. Appl. Math.* **13,** 2, 247–250, May 1960.

An approximate calculation of the heat transfer by convection from a heated, rotating hemisphere is presented for the case of an arbitrary distribution of surface temperature along a meridian, the surface temperature distribution being axisymmetrical with respect to the axis of rotation. Numerical results are compared with results of Davies for the heat transfer from a rotating disk, wherefrom it is shown that the effect of curvature is to decrease the heat-transfer rate.

R. M. Drake, USA

1603. Scala, S. M., and Baulknight, C. W., Transport and thermodynamic properties in a hypersonic laminar boundary layer: Part 2, Applications, *ARS J.* **30,** 4, 329–336, Apr. 1960.

This second part of a two-part paper presents a method of solution of the boundary-layer equations using (1) transport and thermodynamic properties for high-temperature dissociated air as determined by the methods given in the previously published first-part [AMR 12(1959), Rev. 4086] and (2) the Curtiss Hirschfelder and Bird method of predicting thermal conductivity of a chemically reacting gas. Digital computer calculations of (a) boundary-layer profiles obtained at the stagnation point, (b) gas properties as a function of local temperature in the boundary layer, and (c) hypersonic stagnation point heat transfer as a function of environmental conditions for variable Prandtl and Lewis numbers are given. The latter are also compared with earlier theoretical results and with experimental data. The authors' stagnation heat-transfer curves fall within the limited experimental shock-tube data points of Rose and Stark at low altitude and fall considerably below the high-altitude shock tube data of Vitale, et al. The scatter of the data is of the same order as differences in results predicted by various theories and no conclusion can be obtained concerning the superiority of the authors' theoretical treatment over any other.

R. E. Bolz, USA

1604. Hodge, R. I., A turbine nozzle cascade for cooling studies: Part 1, The measurement of mean Nusselt numbers at the blade surface, Aero. Res. Counc. Lond. Curr. Pap. 492, 15 pp. + 10 figs., 1960.

A description is given of a cascade tunnel designed and built to evaluate various cooling systems for turbine blading. Details of the tunnel instrumentation and operating conditions are included.

Gas-to-blade-surface mean-heat-transfer coefficients have been determined in tests using a special blade. These experimental coefficients are related to blade Reynolds number and to the temperature ratio between gas and blade surface.

The second part to the present pair of Memoranda compares these experimental results with predicted values.

From author's summary

1605. Bernard, J. J., and Siestrunck, R., Heat transfer in separated flow (in French), Advances in Aeronautical Sciences, Vol. 1 (Proc. of the First International Congress in the Aeronautical Sciences, Madrid, Sept. 8–13, 1958), Pergamon Press, 1959, 314–332.

Frictional and kinetic heating due to high stagnation temperatures on the surface of objects in high-speed flight has been found under some conditions to be unexpectedly low past the flow separation point. Certain regions of a missile's surface may thus be protected against excessive heating if flow separation is induced upstream. In some configurations, however, the opposite effect has been observed: the heat flux flowing across the wake behind a flow separation point may actually be greater than that flowing across a boundary layer attached to the wall under otherwise similar conditions of main stream velocity, upstream configuration, etc. The present paper seeks to clarify this confusing phenomenon by examining some experimental results of others and citing some theoretical approaches. No clear-cut pattern emerges. As one possible reason for the apparent contradiction of these experimental observations, the authors cite the radial temperature variation in rapidly spinning vortices.

A. W. Gessner, USA

1606. Rogers, G. F. C., and Mayhew, Y. R., Evaluation of bulk velocity and temperature for turbulent flow in tubes, *Inter. J. Heat Mass Transfer* **1,** 1, 55–67, June 1960.

Methods are described for determining the bulk (spatial mean) velocity and temperature in fully developed turbulent flows in smooth tubes. The location of the mean velocity or temperature point from the wall of the tube or the determination of the bulk velocity or temperature in the tube from the measurement of the maximum values depends in the analysis on the universal velocity $u(\tau_w/\rho)^{-1/2} = f[y(\tau_w/\rho)^{1/2}/\nu^{-1}]$ and the universal temperature law of H. B. Squire for various Prandtl numbers.

It is found that for pipe Reynolds numbers beyond 5×10^3, the location of the mean velocity and temperature point from the wall

of the pipe does not change with Reynolds numbers. This conclusion was found not to apply, however, to mean temperature point for flows with Prandtl numbers less than 0.1.

Charts are given for determining the bulk velocity and temperature from the measured maximum values at the center of the tubes.

Finally a method is presented for correcting stream temperatures when measured with thermocouples that extend radially in the hot stream. The errors calculated are from conduction on the junction wires from the rest of the stream. S. Eskinazi, USA

1607. Tao, L. N., On combined free and forced convection in channels, *ASME Trans.* **82** C (*J. Heat Transfer*), 3, 233–238, Aug. 1960.

A mathematical approach for the analysis of free and forced convection by a fully developed laminar flow in a vertical channel. The axial wall temperature gradient is considered constant. The method may be employed for the cases with or without heat generation. The author claims that the method is more direct than those used by previous investigators. The analysis has been applied to the cases of flow between parallel plates and in rectangular channels. Studies of other configurations are being investigated.

G. A. Hawkins, USA

1608. Leslie, F. M., Free convection in the tilted open thermosyphon, *J. Fluid Mech.* **7**, 1, 115–127, Jan. 1960.

Lighthill analysis of a vertical thermosyphon is extended by considering the external acceleration to be at a small angle to the axis. The problem is similar to that of free convection in rotating turbine blades; the component of the buoyancy force perpendicular to the axis simulates closely the Coriolis force on the moving fluid. Only laminar flow is considered. Approximate solutions are found, one for large values of Rayleigh numbers when the boundary layer does not fill the tube and another for small Rayleigh numbers when the boundary layer fills the tube.

It is found that as the inclination of the external acceleration to the axis is increased the boundary-layer thickness decreases, giving an increase in heat transfer, a result which is in agreement with unpublished experimental results. The analysis does not predict an initial decrease in the Nusselt number (as observed in experiments) because it does not allow for instabilities in the boundary layer. N. Zuber, USA

1609. Roberts, L., On the melting of a semi-infinite body of ice placed in a hot stream of air, *J. Fluid Mech.* **4**, 5, 505–528, Sept. 1958.

Problem may, perhaps, be better described as steady melting and ablation at stagnation point of blunt body. No vaporization is allowed. Complete Navier-Stokes equations are solved with compressibility accounted for in air phase. Exact boundary conditions are satisfied at air-liquid and liquid-solid interfaces. Author points out that convection in liquid layer shields solid thermally, thereby reducing melting rate. Throughout solution ratio of air density to liquid density is considered small, and results are expanded in this parameter. Final equations are, nevertheless, extremely cumbersome, and reviewer has doubts that game is worth the candle. Application would undoubtedly be to metallic body (meteor, reentering missile) for which Prandtl number of liquid is small and shielding effect consequently negligible. Also reviewer disagrees with statement "melting of a body of finite length has no steady solution." Reviewer has, in fact, studied melting and ablation of finite body [Proc. 3rd U. S. Nat'l Congr. Appl. Mech., 1958; AFOSR TN 58-789, Aug. 1958] and obtained steady solution for steady input.

T. R. Goodman, USA

1610. Roberts, L., Mass transfer cooling near the stagnation point, NASA TR R-8, 16 pp., 1959.

Fluid injection of foreign gas (coolant) at stagnation point of airborne vehicle is studied. An integral method is used which renders analysis considerably more tractable than that of preceding review. Author presents explicit method for calculating rate of mass injection necessary to achieve desired wall temperature for given stagnation and coolant temperatures. Analysis reveals importance of coolant with high specific heat, low Schmidt number and high Prandtl number. (See AMR **12**(1959), Rev. 3074 for author's similar conclusion with respect to subliming solid.)

T. R. Goodman, USA

1611. Roberts, L., A theoretical study of stagnation-point ablation, NASA TR R-9, 11 pp., 1959.

See AMR **12**(1959), Rev. 3074.

1612. Roberts, L., Stagnation-point shielding by melting and vaporization, NASA TR R-10, 24 pp., 1959.

In title problem, author continues his analysis of stagnation point cooling by ablation. (See preceding reviews.) Integral method is used again, and now author comes to same conclusion as reviewer, viz., that liquid layer contributes negligible heat shielding. No new qualitative conclusions are reached concerning desirable properties for an ablating material, but the quantitative analysis is presented in detail. This set of reports constitutes an important contribution to the technology of mass transfer cooling.

T. R. Goodman, USA

1613. Trussell, D. H., and Weidman, D. J., A radiant heater to simulate aerodynamic heating in a wind tunnel, NASA TN D-530, 21 pp., Nov. 1960.

A recently developed radiant-heating test technique for simulation of aerodynamic heating in a wind tunnel is described. The heating device, which utilized quartz-tube lamps, was operated successfully while exposed directly to a supersonic airstream. Tests were made on a calibration panel, and experimental temperature and pressure data are presented. Results indicate that initial heating rates of about 26 Btu/(sq ft)(sec) are obtainable at a distance of 12 inches from the heater. Further applications of the basic design are discussed briefly.

From authors' summary

1614. Bird, G. A., The effect of thermal radiation on the inviscid hypersonic flow over a blunt body, *J. Aerospace Sci.* **27**, 9, 713–714 (Readers' Forum), Sept. 1960.

1615. Holman, J. P., and Boggs, J. H., Heat transfer to Freon 12 near the critical state in a natural-circulation loop, *ASME Trans.* **82** C (*J. Heat Transfer*), 3, 221–226, Aug. 1960.

An experimental study of the heat transfer to Freon 12 in the critical region by use of a closed natural-circulation loop. The program covered temperature and pressure ranges of 150 to 400 F and 500 to 950 psi respectively. Data are presented in dimensionless correlations. One conclusion was that there is no appreciable increase in heat-transfer coefficient in the critical region over that experienced in the superheat region.

G. A. Hawkins, USA

1616. Nishikawa, K., and Yamagata, K., On the correlation of nucleate boiling heat transfer, *Inter. J. Heat Mass Transfer* **1**, 2/3, 219–235, Aug. 1960.

Authors derive an equation for the correlation of nucleate boiling heat-transfer data for fluids of different physical properties and applicable to forced-convection saturated boiling or surface boiling. The predictions of the equation are compared with the results of experiments of the authors and previous work, in graphical form, without specific reference to the precentage of accuracy.

The form of the equation is derived on the basis of a physical

model of the bubble dynamics rather than by the use of dimensional analysis. It is the authors' claim that the analysis of elementary processes in boiling is preferable to the deduction of correlation parameters by dimensional reasoning. While this would be accepted in principle by most readers, the particular model analysis carried through by the authors is not wholly convincing.
L. M. Grossman, USA

1617. Friedman, A., Free boundary problems for parabolic equations: Part 3, Dissolution of a gas bubble in liquid, *J. Math. Mech.* **9,** 3, 327–345, May 1960.

A continuation of work reviewed in AMR **13**(1960), Rev. 5361, studying the growth and decay of gas bubbles in a liquid. It is shown that even supersaturated solutions are stable in the sense that bubbles cannot grow in them if they are below a minimum initial size.
From author's summary by G. Horvay, USA

1618. Grigor'ev, L. N., and Usmanov, A. G., Heat transfer during boiling of binary mixtures, *Soviet Phys.–Tech. Phys.* **3,** 2, 297–305, Nov. 1958. [Translation of *Zh. Tekh. Fiz., Akad. Nauk SSSR* **28,** 2, 325–332, Feb. 1958 by Amer. Inst. Phys., Inc., New York, N. Y.].

1619. Spalding, D. B., A standard formulation on the steady convective mass transfer problem, *Inter. J. Heat Mass Transfer* **1,** 2/3, 192–207, Aug. 1960.

Author derives a consistent "standard" formulation for calculating steady-state convective mass transfer, suitable for chemical engineering, aeronautics, air-conditioning, etc. Mass-transfer velocity ("flux") is the product of a surface conductance g, dependent on fluid flow factors, and a dimensionless driving force B, which uses mass fraction as the unit of concentration.

Equations for g are given for mass transfer through laminar and turbulent boundary layers, and for B for transfer of an inert substance, and of a substance with simple or complex chemical reactions, and for enthalpy transfer. The method is illustrated by the process of fuel vaporization and/or combustion under various conditions.
C. F. Bonilla, USA

1620. Dzung, L. S., A cooling problem of pebble-bed nuclear reactors, *Inter. J. Heat Mass Transfer* **1,** 2/3, 236–241, Aug. 1960.

In gas-cooled nuclear reactors with uniform coolant channels nonuniformity in heat generation causes unfavorable coolant distribution. With pebble-bed reactors the maldistribution is yet worse. Author gives the fundamental equations of gas flow through a porous bed with nonuniform heat generation. In general, digital computer solutions only are feasible. However, the analytical solution is derived for a cylindrical "bare reactor" with J_0 radial and cosine axial heat generation, laminar flow with low fractional pressure drop, and constant viscosity. For a mixed mean outlet absolute temperature T_2 double T_1, the maximum local (axial) $T_3 = 5.15\ T_1$; for $T_2/T_1 = 3$, $(T_3)_{max} = 14.7\ T_1$. The further increase due to film temperature drop, and decrease due to heat conduction, is neglected.
C. F. Bonilla, USA

1621. Spalding, D. B., Heat and mass transfer in aeronautical engineering, *Aero. Quart.* **11,** 2, 105–136, May 1960.

Author suggests that the available techniques for dealing with heat and mass transfer problems, which have been used for many years in mechanical and chemical engineering, are still only imperfectly exploited in aeronautical engineering, partly because of differences of language and notation. The present paper is intended to demonstrate general and simple methods capable of handling aeronautical problems as special cases, and also to show that these methods still need further development of a sort that aerodynamicists may be best fitted to provide. The examples considered involve the laminar boundary layer at the nose of an axisymmetric body at hypersonic flight speed and the turbulent boundary layer in the nozzle of a rocket motor. The paper is mainly expository in nature.
D. W. Dunn, Canada

1622. Gadd, G. E., A note on the theory of the Stanton tube, Aero. Res. Counc. Lond. Rep. Mem. 3147, 10 pp., 1960.

Existing theories for the Stanton tube are critically reviewed, and the paper outlines a simple method which predicts the calibration function at high Reynolds numbers to the right order of magnitude.
From author's summary by A. J. Chapman, USA

1623. Hodge, R. I., A comparison between the measured and predicted cooling performance of an internally spanwise ventilated turbine nozzle blade, Aero. Res. Counc. Lond. Curr. Pap. 494, 22 pp. + 13 figs., 1960.

Profile average surface temperature at midspan and over-all coolant pressure drop have been measured previously in a tunnel examination of a particular air-cooled turbine nozzle section. In this paper values estimated using a previously developed procedure are compared with those test results.

The agreement in the comparison is fairly good except for the effect of the gas/coolant temperature ratio. It is suspected that the internal heat transfer equation adopted in the earlier procedure is not applicable at the low Reynolds numbers of the coolant flows in this case.
From author's summary

Combustion

(See also Revs. 1220, 1396, 1528, 1619, 1633, 1638, 1642)

Book—1624. Sitkei, G., Mixture formation and combustion in diesel engines [A keverekképzes es eges lefolyasa Diesel-motorokban], Budapest, Academy Publishing Co., 1960, 206 pp. + 128 figs. + 7 tables + 66 refs. 50 Hungarian Forint ($4.50)

A detailed, mainly theoretical and mathematical treatise on subject is based on previous research work in several countries, including the original work of the author and his coworkers; these are listed in the bibliography, about half of which refer to Soviet research. Some experimental research is also included, mainly in the form of graphs and tables; a few design drawings illustrate the realization of concepts and principles. Subject is treated under the following headings: I. The injection process (general considerations, calculation of velocity and pressure phenomena with open and closed nozzle, energy expended and utilized in injection, influence of design parameters on the injection characteristics, effect of injection piping, application of the principle of similitude). II. Theory of atomization (causes and forms of jet breakup, external and internal forces, dynamics of drops, penetration, cone angle, distribution of fuel in the spray). III. Evaporation (heat transmission to droplets, characteristics of evaporation and diffusion, influence of air flow, evaporation of fuel and its effect on the working cycle). IV. Diesel combustion (general principles, self-ignition phenomena in mixtures of hydrocarbon and air, effect of physical and chemical characteristics of fuel on the ignition and combustion process, combustion stages, ignition lag, propagation of combustion, indicator diagram, dynamics of cycle). V. Mixture formation in various combustion chambers (general problems of mixture formation, combustion chambers for direct injection, swirl chambers, pre-combustion chambers, calculation of flow phenomena, energy relations and energy balance during mixture formation, heat balance).

The author has rendered a valuable service to research and development engineers concerned with further improvements of the

diesel engine and with its adaptation for various fuels and applications. It would be desirable if the complicated process of pressure and velocity surges in the injection piping and nozzle were treated not only mathematically—which is difficult to follow—but also graphically, which is far clearer and more comprehensible. More numerical examples would also contribute to the practical value of this work. But these are only minor shortcomings of an otherwise excellent treatise. In the opinion of this reviewer the book's high scientific level, and comprehensive coverage of subject, would well merit its translation into one of the western languages whereby its usefulness for research men in the western countries would be enhanced.

K. J. DeJuhasz, USA

1625. Turcotte, D. L., Stable combustion of a high-velocity gas in a heated boundary layer, *J. Aero/Space Sci.* **27,** 7, 509–516, July 1960.

Difference between two combustion types in boundary layers is explained. Present experimental work treats phenomenon that heated wall forms continuous ignition source for combustion in turbulent boundary layer. It is found that combustion process is essentially different from that in unheated boundary layers where wall-quenching of flame occurs. In present set-up, ignition point appeared to be stable and well defined in accordance with physical picture. Measured activation energy required for ignition is in rough agreement with available estimates.

F. van der Walle, Holland

1626. Gross, R. A., and Chinitz, W., A study of supersonic combustion, *J. Aero/Space Sci.* **27,** 7, 517–524, 534, July 1960.

Steady, stable, plain and oblique detonation waves were created in air-hydrogen and air-methane mixtures in high-temperature, steady-flow supersonic tunnel. Ignition conditions and properties across wave were measured. Local-wave fluid-dynamic properties agree well with detonation theory. Detonations appear to be steady and reproducible. Unsteadiness existed only with combustion in wall boundary layer. Hysteresis effect was found, i.e. combustion could be sustained after total temperature was decreased below ignition temperature; however, origin of it is not yet clear. Experimental data are presented and compared with other studies and theory. Experimental behavior of detonations and their possible utility are discussed.

F. van der Walle, Holland

1627. Hirschfelder, J. O., Diffusion coefficients in flames and detonations with constant enthalpy, *Physics of Fluids* **3,** 1, 109–112, Jan./Feb. 1960.

The assumption of constant specific enthalpy of material in passing through a flame is examined analytically, and it is found that the assumption is equivalent to setting all the coefficients of binary diffusion equal to each other and their corresponding Lewis numbers equal to unity. It is noted that these restrictions are unrealistic in some combustion systems.

E. W. Price, USA

1628. Cheng, S.-I., Combustion instability in solid rockets using propellants with suspended metallic powders, AFOSR TN 59-1002 (Princeton Univ., Dept. Aero. Engng. Rep. 482), 32 pp., Sept. 1959.

Author extends his treatment of combusion instability in solid and liquid propellant rockets [see *Jet Propulsion* **24,** p. 37 and 102, 1954; AMR **8**(1955), Rev. 841, and AGARDograph no. 8, 1956], where the metallic powder acts as a perturbation, thereby leaving the physical and chemical kinetic characteristics of the propellant unchanged. The metallic particles are thought of as heat sources only and their effect on the mean temperature and density is treated in a three-dimensional model under the limiting assumption that the strength of the heat sourses is time-independent, and in a one-dimensional model, where the limiting assumption is dropped. The result of model I is that the metallic powder seems to have a stabilizing effect (but not very significant) and the result of model II is that unstable oscillations may be introduced which are, in general, out of phase with the oscillations from the combustion of the basic propellant.

C. B. Ludwig, USA

1629. Barduhn, A. J., Patel, B. S., Meyer, W., and Smura, B. B., Adsorption of halogenated fire-extinguishing agents on powders, NASA TR-R-51, 41 pp., 1960.

Report describes theory and some tests of a new idea for a fire-extinguishing agent. A combination of halogenated methanes adsorbed on fine powders is used to provide a single agent which combines powder and vapor for fire fighting. Fundamental adsorption data are given for four Freons on carbon, silica gel, alumina and molecular sieves. Adsorptions up to 100% of their own weight were obtained for some agents. Tests using a standard dry powder extinguisher showed no significant improvement. Authors recommend development of extinguisher designed specifically for use with adsorbed systems and predict an enhanced fire-fighting effectiveness.

R. A. Gross, USA

1630. Spalding, D. B., Tracer studies of mixing in the wake of flame holders, Tiltman Langley Limited, Redhill Aerodrome, England, Tech. Note no. 2 (Contract AF 61(514)-1213), 50 pp., Nov. 1959.

The wake regions of four axially symmetrical flame-holders have been studied by the injection of a tracer (argon) at one location and the analysis of the tracer concentration at a second location. Both injection and sampling locations were varied so as to provide comprehensive description of mixing patterns in terms of influence coefficients, surveys being carried out when the gas stream was pure air (cold flow) and when the gas stream was a propane-air mixture burning behind the baffle (hot flow). The influence coefficients were used to enable predictions to be made of the flame-holding characteristics of the baffles, in terms of their dimensionless "chemical loading factor" for extinction.

It was demonstrated that the values of the influence coefficients, and the sizes of the recirculation zones, were appreciably larger for the hot flow than for the cold. Loading factors predicted from the hot-flow influence coefficients are therefore an order of magnitude greater than those based on the cold-flow coefficients.

From author's summary

Prime Movers and Propulsion Devices

(See also Revs. 1220, 1560, 1624, 1628)

1631. Wanhainen, J. P., Ross, P. S., and De Witt, R. L., Effect of propellant and catalyst bed temperatures on thrust buildup in several hydrogen peroxide reaction control rockets, NASA TN D-480, 25 pp., Oct. 1960.

An investigation was undertaken to determine the effect of chamber and propellant feed temperatures on the starting characteristics of hydrogen peroxide thrust chambers. Start delay times for two types of thrust chamber designs in the 1- to 24-pound-thrust range were obtained over a range of chamber and propellant feed temperatures from 30° to 100°F. Start delay times obtained during the first minute of catalyst bed life and again after 6 minutes of total accumulated running time are presented as a function of chamber and propellant feed temperatures.

The initial cold-start delay time of the hydrogen peroxide thrust chambers investigated was approximately 0.150 second to attain 90% of steady-state chamber pressure at chamber and propellant feed temperatures of 70°F and above. Both thrust chamber designs could be started at chamber and propellant feed temperatures as low as 30°F; start delay times did, however, generally increase at low temperatures. When the chamber was at an elevated temperature from a preceding firing, the start delay time was reduced to approximately 0.050 second, indicating a marked effect of chamber temperature at constant propellant feed temperatures. Accumulated run time affected the starting characteristics only when both the chamber and propellant feed temperatures were at reduced levels. From authors' summary

1632. Cardullo, M. W., Variable-thrust rocket engines and their modes of operation, *J. Aerospace Sci.* **27,** 10, 793–795 (Readers' Forum), Oct. 1960.

1633. Willis, C. M., The effect of catalyst-bed arrangement on thrust buildup and decay time for a 90 percent hydrogen peroxide control rocket, NASA TN D-516, 25 pp., Sept. 1960.

An investigation has been conducted to determine the effects of decomposition-chamber size and catalyst-bed-material arrangement on the thrust buildup and decay time for a 90% hydrogen peroxide control rocket. The rocket produced about 35 pounds of static thrust at a chamber pressure of 300 pounds per square inch. Catalyst-bed loadings ranged up to about 0.50 pound of propellant per second per square inch of bed. Five configurations, including two decomposition-chamber lengths and various arrangements of the catalytic material, were tested on a static test stand at various chamber pressures and rocket-case temperatures. Chamber-pressure-buildup time was mainly dependent upon rocket-case temperature, being long at low temperatures and decreasing to about 0.05 second for rocket-case temperatures equal to or greater than 1000°F. Variations in decomposition-chamber arrangement and steady-state chamber pressure produced little difference in the starting-time delay. Chamber-pressure-decay time to 10% of steady-state chamber pressure varied from about 0.06 to 0.11 second and the chamber pressure decayed completely in 0.08 to 0.20 second at all test conditions.

From author's summary

1634. Levine, J., Martz, C. W., Swain, R. L., and Swanson, A. G., Flight performance of a spin-stabilized 20-inch-diameter solid-propellant spherical rocket motor, NASA TN D-441, 38 pp., Sept. 1960.

A successful flight test of a spin-stabilized 20-inch-diameter solid-propellant rocket motor having a propellant mass fraction of 0.92 has been made. The motor was fired at altitude after being boosted by a three-stage test vehicle. Analysis of the data indicates that a total impulse of 44,243 pound-second with a propellant specific impulse of approximately 185 was achieved over a total action time of about 12 seconds. These results are shown to be in excellent agreement with data from ground static firing tests of these motors. The spherical rocket motor with an 11-pound payload attained a velocity of 15,620 feet per second ($M = 16.7$) with an incremental velocity increase for the spherical motor stage of 12,120 feet per second. From authors' summary

1635. Farley, J. M., and Campbell, C. E., Performance of several method-of-characteristics exhaust nozzles, NASA TN D-293, 33 pp., Oct. 1960.

Nozzle performance data were obtained with three "method-of-characteristics" nozzles and a 15° conical nozzle at pressure ratios up to 130. Each basic configuration was cut off and tested at expansion ratios of 25, 20, 15, and 10. Unheated dry air was used at nozzle inlet pressures up to 22,000 pounds per square foot absolute. Nozzle thrust data were extrapolated to infinite pressure ratio (zero discharge pressure).

As much as 1% increase in thrust, with no increase in nozzle surface area (weight), can be obtained by using a method-of-characteristics nozzle instead of a 15° conical nozzle when operating with a nozzle expansion ratio of 25 and nozzle pressure ratios from 200 to ∞. Conversely, for the same thrust, reductions in nozzle divergent surface area in the order of 25% are possible. The thrust performance of the method-of-characteristics nozzle was not as good as that of the 15° conical nozzle when operating at pressure ratios considerably below design (below 100 for the expansion-ratio-25 nozzles). Theoretical and measured nozzle momentum coefficients agreed within about 0.6%. This is the order of accuracy of both the measured and theoretical values.

From authors' summary

1636. Norton, H. T., Jr., and Keith, A. L., Jr., Effect of base bleed and terminal fairings on the performance of exhaust-nozzle—afterbody combinations at Mach numbers of 1.93, 2.55, and 3.05, NASA TN D-539, 31 pp., Nov. 1960.

An investigation of four exhaust-nozzle—afterbody combinations has been conducted in the Langley 9- by 12-inch blowdown tunnel at Mach numbers of 1.93, 2.55, and 3.05. The models were tested on a pylon-mounted nacelle and the jet exhaust was simulated with cold air. Base bleed was varied from 0 to about 12% of the primary jet weight flow and was discharged into the base region through either a sonic or supersonic bleed nozzle. The models were tested at zero degree angle of attack and the Reynolds number range was from 8×10^6 to 9×10^6 per foot.

The results indicate that the base pressure and the performance of the exhaust-nozzle—afterbody combinations were little affected by the high-velocity base bleed. The efficiency of the terminal-fairing model was only slightly less than that of the convergent-divergent nozzle-afterbody combinations; this difference indicates the loss associated with improved transonic efficiency at higher Mach numbers. From authors' summary

1637. Pitts, W. C., and Wiggins, L. E., Axial-force reduction by interference between jet and neighboring afterbody, NASA TN D-332, 95 pp., Sept. 1960.

Experimental results are presented for an exploratory investigation of the effectiveness of interference between jet and afterbody in reducing the axial force on an afterbody with a neighboring jet. In addition to the interference axial force, measurements are presented of the interference normal force and the center of pressure of the interference normal force. The free-stream Mach number was 2.94, the jet-exit Mach number was 2.71, and the Reynolds number was 0.25×10^6, based on body diameter. The variables investigated include static-pressure ratio of the jet (up to 9), nacelle position relative to afterbody, angle of attack (−5° to 10°), and afterbody shape. Two families of afterbody shapes were tested. One family consisted of tangent-ogive bodies of revolution with varying length and base areas. The other family was formed by taking a planar slice off a circular cylinder with varying angle between the plane and cylinder. The trends with these variables are shown for conditions near maximum jet-afterbody interference. The interference axial forces are large and favorable. For several configurations the total afterbody axial force is reduced to zero by the interference. From authors' summary

1638. Sargent, W. H., and Gross, R. A., Detonation wave hypersonic ramjet, *ARS J.* **30,** 6, 543–549, June 1960.

Performance study is made for a hypersonic ramjet employing a detonation wave combustion process with hydrocarbon fuels. Flight Mach numbers from 2.5 to 10 are considered at an altitude of 130,000 ft. Results include thermodynamic property changes across the detonation wave, air specific impulse, specific fuel

consumption, thermal efficiency, and an analysis of the optimum initial conditions for the detonation wave. Comparison of this engine with subsonic burning ramjet indicates that the latter engine always has better performance, although differences may be small. Analysis differs from those in other cited references in that variable prewave temperatures and strong detonation waves are considered. T. C. Adamson, Jr., USA

1639. Sams, E. W., Performance of nuclear rocket for large-payload, earth-satellite booster, *J. Aero/Space Sci.* **27,** 7, 481-492, 516, July 1960.

The author has calculated the ratio of payload to initial weight as well as several other significant parameters for a nuclear rocket which is to place a payload in a 300-mile orbit. The rocket is of the heat-transfer type and a single-stage vehicle is assumed. Two different reactor types are considered. The core of the first is made of graphite containing fuel in dispersed form and the core of the second is made of tungsten-uranium fuel elements with BeO as a moderator. The analysis emphasizes the heat-transfer and friction processes within the reactor. It is concluded a payload-to-weight ratio of approximately 0.10 could probably be achieved with either rocket.

Considerations regarding cost and engineering feasibility are beyond the scope of the paper. Nevertheless, the reviewer feels that the reader should keep such aspects clearly in mind when comparing the two types of nuclear rockets which are discussed or when comparing the performance of the nuclear rocket to that of a chemical unit. R. H. Sabersky, USA

1640. Reynolds, T. W., and Childs, J. H., A graphical method for estimating ion-rocket performance, NASA TN D-466, 27 pp., Aug. 1960.

Equations relating the critical temperature and ion current density for surface ionization of cesium on tungsten are derived for zero and finite electric fields at the ion-emitting surface. Equations obtained are used to construct graphs which can be used for solving problems related to ion-rocket performance. Effect of operation at less than space-charge limited current density and of nonuniform propellant flux onto the ion-emitting surface are also treated.

Equation relating emitter temperature to space-charge-limited current density on which performance charts are based was established from experimental data obtained with pure tungsten ion emitters in small-scale apparatus. NASA data [Dangle, E. E. and Lockwood, D. L., "NASA experimental research with ion-rockets," Amer. Rocket Society Meeting, Los Angeles, May 1960] indicate that higher emitter temperatures are required for engine described in reference than those indicated by derived equation. Discrepancy is attributed to surface impurities on engine ion-emitter. Impurities may alter thermal emissivity of the emitter from those of pure tungsten—resulting in radiation power losses greater than those computed in subject article.

From authors' summary by S. Lampert, USA

1641. Goethert, B. H., and Taylor, D., High altitude testing of propulsion systems (in English), *Z. Flugwiss.* **8,** 7, 202-211, July 1960.

The test requirements placed upon engine test facilities by high performance aircraft and missile propulsion systems are discussed. The exhaust system performance is shown to be the limiting factor in most ground test facilities. Various simple methods of augmenting the exhaust system performance sufficiently to meet the stringent requirements are shown, and some test results are presented. These methods are in use at the Arnold Engineering Development Center of the Air Research and Development Command, USAF. From authors' summary

1642. Yaron, A., Some developments of the end-burning charge, *Bull. Res. Council Israel* **8** C, 1, 47-52, Feb. 1960.

Magneto-fluid-dynamics

(See also Rev. 1466)

1643. Kautzleben, H., Considerations of hydromagnetic theory of plasmas (in German), Deutsch. Akad. Wissenschaften, Berlin, Geomagnetisches Inst. Potsdam, no. 20, 11-87, 1958.

Author examines at length the fundamentals underlying plasma physics. Previous literature on subject is evaluated. Although a kinetic treatment would in principle adequately describe plasma flow, author feels this is too laborious. Therefore he investigates simpler methods of approach. In particular, criteria for treating plasma physics from the macroscopic or hydromagnetic point of view are discussed. A. L. Loeffler, Jr., USA

1644. Kogan, M. N., Magnetodynamics of plane and axisymmetric flows of a gas with infinite electrical conductivity, *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **23,** 1, 92-106, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Author investigates flows in which the magnetic field is parallel to the velocity of the oncoming flow. The type of flow (elliptic or hyperbolic) depends not only on the flow Mach number M_∞, but also on a parameter N_∞^2, the ratio of magnetic and hydrodynamic pressures. There appear two flow regimes not encountered in ordinary gas dynamics: a quasi-hyperbolic regime which occurs at subsonic velocities, in which there may be shock waves, inclined upstream, and an elliptic type which occurs at supersonic speeds, in which only *strong* shock waves occur. The properties of shock waves, characteristics and simple waves are investigated. The results are used to discuss possible flows around finite bodies (plane and axisymmetric). Particular attention is paid to the quasi-hyperbolic regime in which disturbances can penetrate upstream, and flow patterns look like mirror images of ordinary supersonic flows.

This is the first of a series of papers by the author on magneto-hydrodynamic flows over bodies of the type discussed in conventional gas dynamics; the area of interest is similar to that covered in the papers of Sears, Resler and McCune (e.g. J. E. McCune and E. L. Resler, Jr., "Compressibility effects in magnetoaerodynamic flows past thin bodies," *J. Aero/Space Sci.* **27,** p. 493, 1960]. A. Roshko, USA

1645. Germain, P., On certain types of flow of perfectly conducting fluids (in French), *Rech. Aéro.* no. 74, 13-22, Jan./Feb. 1960.

The problem treated is a study of the effects of magnetic fields in the direction of flow in an electrically conducting steady perfect fluid. This work is a broader approach to the problem recently studied by M. N. Kogan.

For relatively weak magnetic fields, it is found that the flow field has the same properties as those encountered in classical gas dynamics. However for stronger magnetic fields, the flow field may show subsonic régimes with real characteristics or supersonic régimes with imaginary characteristics. The problem becomes one of knowing how to orient these characteristics. The qualitative study of propagation of unsteady perturbations shows that in the case of subsonic flow with real characteristics, such characteristics are oriented in the upstream direction.

S. Eskinazi, USA

1646. Devienne, F. M., and Souquet, J., Production of a molecular beam at a very high speed and relatively great density—Realization of a molecular gun (in English), AFOSR TN 60-906,

Laboratoire Mediterraneen de Recherches Thermodynamiques, 30 pp. + figs., June 1960.

Purpose was to obtain molecular beams whose intensity exceeds 10^{15} molecules per square centimeter per second, with speeds ranging between 10 and 30 Km per second, or higher.

The cylindrical ionic beam is obtained by the high frequency ionization of a molecular beam coming from a hypersonic nozzle. This beam passes through a quartz tube around which a coil is wound. A high frequency current flows through the coil. The ionization efficiency exceeds 10%.

The ions are separated from the molecules by means of a magnetic field generated by an electromagnet. Then the ion beam is accelerated and brought to the satisfactory voltage by a linear accelerator; it then crosses a molecular beam coming out of another hypersonic nozzle. In the interaction space of the two beams, charge or momentum exchanges are produced and after deflection of ions and pumpage of slow molecules, a beam of high-speed molecules in the initial direction of the ion beam is obtained.

The molecules belonging to the molecular beam are detected by means of a tungsten strip heated at a relatively low temperature. The speed is measured with a special device.

The preliminary researches carried out in a vacuum tank resulted in a beam of 3.10^{15} molecules per square centimeter per second.

The apparatus described permits the study of all the interaction phenomena between a body moving at a cosmic speed and a very rarefied gas in free molecular flow regime.

From authors' summary

1647. Zhigulev, V. N., Theory of the magnetic boundary layer, *Soviet Phys.-Doklady* **4,** 1, 57-60, Aug. 1959. (Translation of *Dokladi Akad. Nauk SSSR (N.S.)* **124,** 5, 1001-1004, Jan./Feb. 1959 by Amer. Inst. Phys., Inc., New York, N. Y.)

Author develops the boundary-layer equations for the steady flow of a conducting fluid past a semi-infinite plate when a current flows in the plate (1) parallel to its edge and (2) perpendicular to its edge. The Reynolds number and the magnetic Reynolds number are assumed to be large. Possible forms of similarity solutions are discussed. M. B. Glauert, England

1648. Tsorkovnikov, Yu. A., The question of convectional instability of a plasma, *Soviet Phys.-Doklady* **5,** 1, 87-90, July/-Aug. 1960. (Translation of *Dokladi Akad. Nauk SSSR (N.S.)* **130,** 2, 295-298, Jan./Feb. 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

The convectional instability of a plasma, i.e., instability which arises due to spatial inhomogeneities of the parameters by which it is characterized, has been discussed in terms of the kinetic equations, neglecting collisions between particles. Such an approach is reasonable in the investigation of a low-density plasma, although the resultant equations are highly complicated. In the present paper, author considers the same problem, proceeding, however, from the simpler hydrodynamic equations. In solving the equations, author restricts himself to the study of the behavior of small-scale perturbations, neglecting the boundary conditions, a justifiable approach in the study of convectional instability.

From author's summary

1649. Pai, S. I., The wave motions of small amplitude in a fully ionized plasma, Part 1, Without external magnetic field, AFOSR TN 59-1271 (Inst. Fluid Dynam. Appl. Math., Univ. Maryland TN BN-191), 24 pp., Nov. 1959.

Linearized study of one-dimensional transverse and longitudinal waves is based on classical two-fluid scheme together with assumptions that both ions and electrons are inviscid non-heat-conducting perfect gases, and that interaction forces between the two gases are proportional to the difference between their mean velocities. Classical results for ideal plasmas (infinite conductivity) are reobtained; also effects of finite conductivity on damping are indicated. R. Vaglio-Laurin, USA

1650. Pai, S. I., The wave motion of small amplitude in a fully ionized plasma, Part 2, With longitudinal applied magnetic field, AFOSR TN 60-576 (Inst. Fluid Dynam. Appl. Math., Univ. Maryland TN BN-207), 28 pp., Apr. 1960.

Continuation of linearized analysis in preceding review and based on same scheme and assumptions. Classical results on interaction of basic modes of transverse waves in an ideal plasma in presence of external magnetic field are reobtained; also effects of finite conductivity on these waves are indicated.

R. Vaglio-Laurin, USA

1651. Landshoff, R. K. M., Magnetohydrodynamic waves, Missiles & Space Div., Lockheed Aircraft Corp., Sunnyvale, Calif., Rep. LMSD-288269, 20 pp., 1960.

Author presents an adequate comprehensive and analytic treatment of magnetohydrodynamic waves. Basic equations are written down and usual linearization techniques outlined.

Wave propagation modes with no dc magnetic field, with dc field (Alfvén waves) and in an anisotropic medium (crystal or plasma in magnetic field) are clearly outlined.

Report concludes by discussing in some detail the effects of high-frequency behavior as the frequency approaches the lowest ion gyro-frequency and the consequent changing dispersion. Thirteen excellent and recent references accompany the survey.

R. Culver, Australia

1652. Sturrock, P. A., The excitation of plasma oscillations, AFCRC-TN 59-891 (Stanford Univ., Microwave Lab. Rep. 610), 12 pp., Sept. 1959.

The theory of Bohm and Gross and the experiments of Looney and Brown upon the excitation of plasma oscillations by the two-stream mechanism, which appear superficially to be in disagreement, are shown to be compatible with each other and with related experiments. From author's summary

1653. Clauser, F. H., Plasma dynamics, AFOSR TR 59-125 (Johns Hopkins Univ., Dept. Aeronautics), 75 pp., Aug. 1959.

Paper gives a stimulating and interesting discussion of a one-dimensional theory of an electron gas. As the author remarks, much of the current study of plasma dynamics proceeds on the assumption that electrical charges of opposite size are present in equal quantity so that the medium has no net electric charge. He developed his theory of the pure electron gas "in order to see what lies behind this assumption and what is overlooked in making it." It would seem that the notion of net electric neutrality does not need justification; it is further not apparent how the present paper contributes to this point. A specific comment should be made regarding the treatment of the electron gas at the absolute zero of temperature. This treatment is developed in an entirely classical manner whereas it is well-known that the quantum mechanical effect of degeneracy is decisive at and near $T = 0$.

M. S. Plesset, USA

1654. Zhigulev, V. N., Theory of electrical discharge in a moving conducting medium, *Soviet Phys.-Doklady* **4,** 1, 61-64, Aug. 1959. (Translation of *Dokladi Akad. Nauk SSSR (N.S.)* **124,** 6, 1226-1229, Jan./Feb. 1959 by Amer. Inst. Phys., Inc., New York, N. Y.)

Author obtains solutions for the steady flow of incompressible conducting fluid past a point or line source of electric current. The current flow downstream in a layer of thickness proportional to the inverse square root of the magnetic Reynolds number, which

is assumed to be large, and Joule heating produce high temperatures in the layer. M. B. Glauert, England

1655. Balescu, R., Irreversible processes in ionized gases, *Physics of Fluids* **3,** 1, 52–63, Jan./Feb. 1960.

The diagram technique developed by Prigogine and the author [*Physica* **25,** p. 281, p. 302, 1959; AMR **13**(1960), Rev. 4802] is applied to a gas of electrons moving in a compensating uniform background of positive charge. The resulting equations show the existence of "charge clouds" moving with the electrons, and also the presence of collective effects leading to nonlinear effects.

D. Ter Haar, England

1656. Sorokin, V. S., and Sushkin, I. V., Stability of equilibrium of a conducting liquid heated from below in a magnetic field, *Soviet Phys.-JETP* **11,** 2, 440–445, Aug. 1960. (Translation of *Zh. Eksp. Teor. Fiz., USSR* **38,** 612–620, Feb. 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

The general effect of a uniform magnetic field on the stability of the equilibrium of a conducting liquid which is heated from below in a cavity of arbitrary shape is investigated. The time variation of the perturbations which arise in the liquid is always monotonic. The critical value of the Rayleigh number C_C^2, above which the equilibrium is unstable, increases monotonically with the Hartmann number M. At small values of M the critical value of the Rayleigh number is proportional to M^2 and the coefficient of proportionality can be computed. The asymptotic nature of the function $C_C(M)$ as $M \longrightarrow \infty$ depends on the shape of the cavity and the direction of the field. From authors' summary

1657. Ludford, G. S. S., Note on airfoil theory in hydromagnetics, Brown Univ., Div. Appl. Math. TR-38, (Contract Nonr-562(07)), 6 pp., Oct. 1960.

1658. Green, T. S., Evidence for the containment of a hot, dense plasma in a Theta pinch, *Phys. Rev. Letters* **5,** 7, 297–300, Oct. 1960.

Aeroelasticity

(See also Revs. 1276, 1322, 1668)

1659. Hsu, P.-T., and Weatherill, W. H., Pressure distribution and flutter analysis of low-aspect-ratio wings in subsonic flow, Mass. Inst. Technol., Aeroelastic Struct. Res. Lab., Tech. Rep. 64-3, 47 pp., June 1959.

This report presents calculations and an extension of the kernel-function approach of P.-T. Hsu, "Flutter of low-aspect-ratio wings: Part 1, Calculation of pressure distributions for oscillating wings of arbitrary planform in subsonic flow by the kernel-function method," Mass. Inst. Technol., Aeroelastic Struct. Res. Lab., Tech. Rep. 64-1, Oct. 1957. Lift and moment in pitch and translation for rectangular (AR = 1, 2) and triangular (AR = 1.07, 2.31) wings compare favorably with other theory and Laidlaw's data. An application to flutter analysis of a 45° swept-delta-wing flutter model is given; the predicted flutter speed is highly conservative but the mode shape data employed are rather suspect.

The extension to include a control surface is given for the two-dimensional case, and the numerical results agree well with previous calculations. The control surface on a three-dimensional planform would appear to introduce several additional complications. A number of typographical errors in the equations of this section have been noted.

H. M. Voss, USA

1660. Rodden, W. P., and Revell, J. D., Oscillatory aerodynamic coefficients for a unified supersonic-hypersonic strip theory, *J. Aero/Space Sci.* **27,** 6, 451–459, June 1960.

The second-order unsteady supersonic flow theory of van Dyke [AMR **7**(1954), Rev. 1185], taking into account the first-order thickness correction and yielding a frequency expansion including the cubic term, has been extended for sweep and finite span effects. The sweep correction has been obtained by introducing the geometrical transformations, exact for the infinite wing. The finite span correction is identified with the known linearized correction inside the tip Mach cone for a swept wing of constant angle of attack in steady flow. Authors justify this by remarking that supersonic flutter occurs at low values of the reduced frequency. Use of this correction to the nonlinear terms is arbitrary, but gives no complication. A. I. van de Vooren, Holland

1661. Hopkins, E. J., Jillie, D. W., and Hicks, R. M., A method of reducing aeroelastic effects of highly swept wings, *J. Aero/Space Sci.* **27,** 8, 634–635 (Readers' Forum), Aug. 1960.

One way of maintaining relatively high lift-to-drag ratios at supersonic speeds is to sweep the wing leading edge to high angles. Author discusses an analytical treatment of an arrow wing designed for $M = 3$ and having a leading edge sweep of 80°. Such highly swept wings tend to unload at the wing tips because of aeroelastic effects. This results in unsatisfactory pitching moments and reduced lift-to-drag ratios. Author shows that these adverse aeroelastic effects can be nearly completely eliminated by warping each wing panel to conform with the curvature of a cylinder having its axis parallel to the flight direction. Warping the wing in this manner would result in no change in local angle-of-attack and only small changes in minimum drag.

A. A. Regier, USA

1662. Johns, D. J., Supersonic flutter of a cylindrical panel in an axisymmetric mode, *J. Roy. Aero. Soc.* **64,** 594, 362–363 (Tech. Notes), June 1960.

Author presents a simple analysis using linear piston theory based on two axially symmetric modes.

J. G. Eisley, USA

1663. Jewell, D. A., A note on hydroelasticity, *J. Ship Res.* **3,** 4, 9–12, Mar. 1960.

AMR **13**(1960), Rev. 2588 indicates that flutter is unlikely for values of "mass ratio" less than 1, and that most hydrofoils have "mass ratios" of about 1/2 as compared with 50 for airfoils. This article points out that in special cases (for example, where a ship participates in the motion) a large "mass ratio" can be obtained. It also points out that existing flutter theory deviates from experimental results when the reduced frequency K is greater than 0.4 and that a typical value of K for a surface ship is 2.0. Other factors peculiar to hydrofoil flutter are noted and the importance of further analytical and experimental investigations is emphasized.

F. E. Reed, USA

1664. Marinescu, A., Vertical vibrations of helicopter rotor blades (in English), *Rev. Mech. Appl.* **4,** 1, 19–27, 1959.

Forced vertical vibrations of rotating helicopter rotor blades are treated using a Rayleigh-type analysis. Untwisted, constant-chord blades, both hinged and cantilevered at the rotation axis, are assumed. Lagrangian methods are used to derive equations of motion in terms of generalized coordinates assumed to be the flexural modes of a uniform nonrotating beam, pinned (or cantilevered) at one end. Generalized forces used in the equations of motion are derived assuming that the aerodynamic forces consist of a steady-state loading with a time-dependent loading superimposed, the time-dependency being assumed sinusoidal in terms of blade azimuth angle. All aerodynamic loading distributions are assumed

triangular. Blade flapping is included in the generalized forces via an equation relating flapping angle and aerodynamic forces.

The resulting equations of motion are linear and uncoupled, hence closed solutions can be written down from which the resonant frequency for each of the assumed modes is derivable.

Results of this analysis permit straightforward calculations of approximate frequencies in flexural modes including aerodynamic forcing effects, for uniform, untwisted rotor blades, either hinged or cantilevered at the rotational axis. Certain coefficients needed, although somewhat involved, can be computed once for all for a given rotor blade. Errors in computed frequencies would presumably increase with increasing rotational speed, and would likely be substantial if method were applied to blades which are twisted to any extent or with mass distributions very much different from uniform. In the latter instances, the results of R. Yntema [NACA TN 3459, 1955], or the method derived by W. Targoff [WADC TR 56-27, December 1955] could be used, among others, to compute more accurate frequencies. Effects of aerodynamic forcing would then have to be determined from an additional analysis.

W. G. Brady, USA

Aeronautics

(See also Revs. 1352, 1442, 1515, 1523, 1525, 1573, 1679, 1685, 1686)

1665. Arens, M., Some requirements for the efficient attainment of range by airborne vehicles, *ARS J.* **30,** 8, 730–734, Aug. 1960.

An effective lift/drag ratio and an effective cruise velocity are derived for a ballistic vehicle and are used to provide a basis for comparison of the range performance efficiencies of ballistic vehicles and of cruising vehicles powered by air-breathing engines. The comparison involves simplifying assumptions regarding the lift/drag ratio and specific fuel consumption of the cruising vehicles. Consideration in general terms is given to the differences in specific weights of the power plants and to the fuel consumed by a cruising vehicle during the initial climb and acceleration. It can be inferred that only for extremely long ranges is the ballistic vehicle likely to prove more efficient.

A. D. Young, England

1666. Stickle, J. W., and Silsby, N. S., An investigation of landing-contact conditions for a large turbojet transport during routine daylight operations, NASA TN D-527, 20 pp., Oct. 1960.

An investigation has been made by the NASA to obtain statistical measurements of landing-contact conditions for a large turbojet transport in commercial airline operations. The investigation was conducted at the Los Angeles International Airport in Los Angeles, California. Measurements were taken photographically during routine daylight operations. The quantities determined were vertical velocity, horizontal velocity, rolling velocity, bank angle, and distance from runway threshold, just prior to ground contact.

The results indicated that the mean vertical velocity for the turbojet-transport landings was 1.62 feet per second and that one landing out of 100 would be expected to equal or exceed about 4.0 feet per second. The mean airspeed at contact was 132.8 knots, with 1 landing in 100 likely to equal or exceed about 153 knots. The mean rolling velocity was about 1.6° per second. One landing in 100 would probably equal or exceed a rolling velocity of about 4.0° per second in the direction of the first wheel to touch. The mean bank angle for the turbojet transports was 1.04°, and right and left angles of bank were about evenly divided. One landing in 100 would be likely to equal or exceed a bank angle of about 3.5°. The mean value of distance to touchdown from the runway threshold was 1560 feet. One landing in 100 would be expected to touchdown at or beyond about 2700 feet from the runway threshold. The mean values for vertical velocity, airspeed, and distance to touchdown for the turbojet transports were somewhat higher than those found previously for piston-engine transports. No significant differences were found for values of rolling velocity and bank angle.

From authors' summary

1667. Mallick, D. L., and Reeder, J. P., Flight evaluation of several spring force gradients and a bobweight in the cyclic-power-control system of a light helicopter, NASA TN D-537, 9 pp., Oct. 1960.

Several feel springs of different rates were evaluated in the power-control system of a light helicopter. In addition, a bobweight and viscous damper for providing maneuvering forces were evaluated. The evaluation was qualitative, based upon the combined opinions of eight research pilots and four nonpilot engineers from NASA. The evaluation revealed that desirable all-around forces for the helicopter were obtained with a ½-lb/in. feel spring for both longitudinal and lateral control combined with a 14-lb/g bobweight. Further investigation proved the necessity of the viscous damper in the bobweight system.

From authors' summary

1668. Kosko, E., Structural analysis of multi-hinged control surfaces, Nat. Res. Counc., Canada, LR-276, 160 pp., Mar. 1960.

An elastic small-deflection analysis is presented of a control surface supported on a structure by means of three or more hinges. Effects of sympathetic flexure, of local bracket flexibility and of angular deflection are taken into account. The existence is shown of an elastic hinge moment, quadratically dependent on the external loading. Application of the method is illustrated by numerical examples, which show that some secondary effects may become more critical than those of direct loading.

From author's summary by G. Isakson, USA

1669. Kuhn, R. E., and Grunwald, K. J., Longitudinal aerodynamic characteristics of a four-propeller deflected slipstream VTOL model including the effects of ground proximity, NASA TN D-248, 136 pp., Nov. 1960.

Results are presented of a wind-tunnel investigation of the longitudinal stability, control, and performance characteristics of a model of a four-propeller deflected-slipstream VTOL airplane in the transition speed range. These results indicate that steady level-flight transition and descending flight-path angles up to 7° or 8° out of the region of ground effect can be accomplished without wing stall being encountered. In general, the pitching moments out of ground proximity can be adequately trimmed by programing the stabilizer incidence to increase with increasing flap deflection, except for a relatively large diving moment in the hovering condition. The deflection of the slipstream onto the horizontal tail in proximity of the ground substantially increases the diving moment in hovering, unless the tail is set at a large nosedown incidence.

From authors' summary

1670. Gillespie, W., Jr., Supersonic aerodynamic characteristics of a low-drag aircraft configuration having an arrow wing of aspect ratio 1.86 and a body of fineness ratio 20, NASA TN D-509, 59 pp., Oct. 1960.

A free-flight rocket-propelled-model investigation was conducted at Mach numbers of 1.2 to 1.9 to determine the longitudinal and lateral aerodynamic characteristics of a low-drag aircraft configuration. The model consisted of an aspect-ratio-1.86 arrow wing with 67.5° leading-edge sweep and NACA 65A004 airfoil section and a triangular vertical tail with 60° sweep and NACA 65A003 section in combination with a body of fineness ratio 20. Aerodynamic data in pitch, yaw, and roll were obtained from transient motions induced by small pulse rockets firing at intervals in the pitch and yaw directions.

From the results of this brief aerodynamic investigation, it is observed that very slender body shapes can provide increased volumetric capacity with little or no increase in zero-lift drag and that body fineness ratios of the order of 20 should be considered in the design of long-range supersonic aircraft. The zero-lift drag and the drag-due-to-lift parameter of the test configuration varied linearly with Mach number. The maximum lift-drag ratio was 7.0 at a Mach number of 1.25 and decreased slightly to a value of 6.6 at a Mach number of 1.81. The optimum lift coefficient, normal-force-curve slope, lateral-force-curve slope, static stability in pitch and yaw, time to damp to one-half amplitude in pitch and yaw, the sum of the rotary damping derivatives in pitch and also in yaw, and the static rolling derivatives all decreased with an increase in Mach number.

Values of certain rolling derivatives were obtained by application of the least-squares method to the differential equation of rolling motion. A comparison of the experimental and calculated total rolling-moment-coefficient variation during transient oscillations of the model indicated good agreement when the damping-in-roll contribution was included with the static rolling-moment terms. From author's summary

1671. Sweet, G. E., Hovering measurements for twin-rotor configurations with and without overlap, NASA TN D-534, 23 pp., Nov. 1960.

Results of an investigation in the Langley full-scale tunnel of the hovering performance of large-scale twin-rotor-helicopter models are presented. Measurements of thrust, torque, and rotor flapping are given for overlapped (approximately 76% of blade radius) and nonoverlapped configurations and for two different rotor solidities. The measured performance is compared with single-rotor measurements and with available rotor theory. These tests show that the hovering performance of a single rotor and of two rotors without overlap or vertical offset are the same and hence may be calculated by single-rotor theory. These tests in conjunction with results of previous coaxial-rotor tests show that the performance of highly overlapped rotors can be reasonably predicted by available rotor theory.

From author's summary

Astronautics

(See also Revs. 1195, 1477, 1639)

1672. Mickelwait, A. B., and Booton, R. C., Jr., Analytical and numerical studies of three-dimensional trajectories to the moon, *J. Aero/Space Sci.* **27,** 8, 561-573, Aug. 1960.

Paper presents conic-section computation of guidance sensitivities for earth-moon coasting trajectories. Results are given in terms of allowable errors in burnout velocity (angle and magnitude) for lunar impact. The three sections of the paper cover, respectively, two-dimensional analysis, three-dimensional analysis, and extension to lunar orbiters.

Principal value of this paper lies in its elucidation of the methods and some results in determining the sensitivity coefficients important for guidance. For example, it is shown how the sensitivity to burnout velocity error reaches its low minimum (at about 36,000 ft/sec at the altitude under study) and how this minimum is affected by three-dimensional considerations.

Lunar orbiters are evaluated in terms of the now familiar Jacobi integral of the restricted three-body problem.

Validity of the analysis is supported by comparison with numerical results. However, there is no implication that it can replace precise numerical procedures for guidance system design, as indeed it cannot.

Reviewer considers this a well-organized and timely paper, valuable particularly for analysts in the trajectory and guidance field. J. Lorell, USA

1673. Pistiner, J. S., Some characteristics of the planar satellite orbit, *ARS J.* **30,** 3, 275-277 (Tech. Notes), Mar. 1960.

Author derives conventional expressions of orbital mechanics. Reviewer was unable to find anything essentially new in the presentation, nor was he impressed with claimed simplicity. H. F. Michielsen, USA

1674. Jacobs, T. A., Penner, S. S., Gill, G., and Eckel, E. F., Approximate theoretical performance evaluation for a diverging rocket (in English), *Astronaut. Acta* **6,** 2/3, 151-158, 1960.

A simplified combustion model, which is motivated by available performance studies on the diverging rocket reactor, has been used as basis for an engine performance evaluation. Comparison with conventional rocket configurations shows that an upper performance limit for the diverging reactor is comparable with performance estimates for engines using an adiabatic work cycle. Development of the diverging reactor for engine applications may, however, offer some advantages for very hot, high-energy, propellant systems. From authors' summary

1675. Pottsepp, L., and Krzywoblocki, M. Z. v., On the application of the stability theory of differential systems to the stability of a missile, Part 1 (in English), *Acta Phys. Austriaca* **13,** 1, 48-64, 1960.

Authors give brief chronological mention of existing information and procede with problem of missile motion stability having rotational symmetry and mirror plane symmetry. Authors indicate that it is possible to find stability criteria without solving motion equations by employing well-known results from stability theory of differential equations. These theories as well as ballistic notations were used whenever it was appropriate. Along with fundamentals are presented matrix notation for systems of equations and some sample solutions. Authors investigate conditions and describe concept of stability, giving theorems and lemmas. Rigid-body motion equations in body-fixed coordinate system are followed by consideration of body possessing angle variation from zero to π, of rotational symmetry and plane of mirror symmetry. Aerodynamic forces are considered. Final sections contain development of normal equations of motion and equations of motion in matrix notation. V. A. Valey, USA

1676. Chapman, D. R., An analysis of the corridor and guidance requirements for supercircular entry into planetary atmospheres, NASA TR R-55, 76 pp., 1959.

Theoretical study of navigation requirements for atmospheric entry of spacecraft limited by maxima of deceleration or heating. Elliptic, parabolic, and hyperbolic orbits are considered for both lifting and pure drag bodies, of arbitrary weight, shape, and size. Results give effect of using aerodynamic lift to widen acceptable entry corridors on Earth, Venus, and show that little can be gained in cases of Mars and Titan. Corridor widths decrease rapidly for supercircular speeds. Finally, author shows that terminal guidance requirements for atmospheric braking are not much more stringent than some already met, provided that vehicle lift and drag can be programmed and that proper sensing is available. The trade-off between such a system and rocket braking is not analyzed here. W. C. Griffith, USA

1677. McGehee, J. R., and Hathaway, M. E., Landing characteristics of a reentry capsule with a torus-shaped air bag for load alleviation, NASA TN D-628, 30 pp., Nov. 1960.

An experimental investigation has been made to determine the landing characteristics of a conical-shaped reentry capsule by

using torus-shaped air bags for impact-load alleviation. An impact bag was attached below the large end of the capsule to absorb initial impact loads and a second bag was attached around the canister to absorb loads resulting from impact on the canister when the capsule overturned. A 1/6-scale dynamic model of the configuration was tested for nominal flight paths of 60° and 90° (vertical), a range of contact attitudes from −25° to 30°, and a vertical contact velocity of 12.25 feet per second. Accelerations were measured along the *X*-axis (roll) and *Z*-axis (yaw) by accelerometers rigidly installed at the center of gravity of the model. Actual flight paths, contact attitudes, and motions were determined from high-speed motion pictures. Landings were made on concrete and on water.

The peak accelerations along the *X*-axis for landings on concrete were in the order of 30*g* for a 0° contact attitude. A horizontal velocity of 7 feet per second, corresponding to a flight path of 60°, had very little effect upon the peak accelerations obtained for landings on concrete. For contact attitudes of −25° and 30° the peak accelerations along the *Z*-axis were about ±15*g*, respectively. The peak accelerations measured for the water landings were about one-third lower than the peak accelerations measured for the landings on concrete.

Assuming a rigid body, computations were made by using Newton's second law of motion and the force-stroke characteristics of the air bag to determine accelerations for a flight path of 90° (vertical) and a contact attitude of 0°. The computed and experimental peak accelerations and strokes at peak acceleration were in good agreement for the model. The special scaling appears to be applicable for predicting full-scale time and stroke at peak acceleration for a landing on concrete from a 90° flight path at a 0° attitude. It appears that the full-scale configuration would have adequate stroke to develop peak accelerations approximately the same as those obtained from the model for the range of attitudes and flight paths investigated.

From authors' summary

1678. Jackson, C. M., Jr., and Harris, R. V., Jr., Static longitudinal stability and control characteristics at a Mach number of 1.99 of a lenticular-shaped reentry vehicle, NASA TN D-514, 16 pp., Oct. 1960.

An investigation has been made in the Langley 4- by 4-ft supersonic pressure tunnel at a Mach number of 1.99 to determine the longitudinal stability and control characteristics of a reentry model consisting of a lenticular-shaped body with two fin configurations (horizontal fins with end plates). Effects of deflecting the larger size fins as pitch-control surfaces were also investigated.

The results indicate that the body alone was unstable from an angle of attack of 0° to about 35°, where it became stable and remained so to 90°. The addition of fins provided positive longitudinal stability throughout the angle-of-attack range and increased the lift-drag ratio of the configuration.

Reducing the horizontal-fin area at the inboard trailing edge of the fin had only a small effect on the aerodynamic characteristics of the vehicle for the condition of no fin deflection. Deflecting the fins appeared to be an effective means of pitch control and had only a small effect on lift-drag ratio.

From authors' summary

1679. Olstad, W. B., A study of the feasibility of inflatable reentry gliders, NASA TN D-538, 41 pp., Oct. 1960.

A study has been made of the feasibility of inflatable reentry gliders. The results of the study indicate that such vehicles appear to be feasible and could be constructed from high-temperature flexible materials which are now available. A typical configuration studied herein had a wing loading of 1.5 pounds per square foot and could operate within an angle-of-attack range from 11° to 66° and at initial entry angles from 0° to −2.5°. The maximum temperature on the wing leading edges for these conditions would not exceed 1300° F. The ratio of structural weight to total weight for the vehicle was about 0.47. With continued development of flexible materials to increase the allowable operating temperature, significant increases in wing loading, payload weight, and structural efficiency can be expected. Some additional features of inflatable gliders are foldability (which alleviates the booster instability problem) and low landing speeds.

From author's summary

1680. Adams, E. W., A comparison of transient and quasi-steady performance of melting-type re-entry shields, *J. Aerospace Sci.* **27**, 10, 791–792 (Readers' Forum), Oct. 1960.

1681. Reismann, H., and Pistiner, J. S., Design and evaluation of a re-entry guidance system (in English), *Astronaut. Acta* **6**, 2/3, 79–114, 1960.

One of the critical problems that must be solved when dealing with manned re-entry is safe recovery of the vehicle and its occupant(s) near a predetermined point on the earth's surface. This paper presents the design and analysis of a fully automatic re-entry guidance system that will permit safe recovery of the vehicle within 0.02 n mi (1 σ) of a preselected point on the earth's surface (neglecting winds and terminal dispersions due to parachute landing) for initial re-entry conditions achievable with existing equipments.

From authors' summary

1682. Gray, V. H., Gelder, T. F., Cochran, R. P., and Goodykoontz, J. H., Bonded and sealed external insulations for liquid-hydrogen-fueled rocket tanks during atmospheric flight, NASA TN D-476, 51 pp., Oct. 1960.

Several currently available nonmetallic insulation materials that may be bonded onto liquid-hydrogen tanks and sealed against air penetration into the insulation have been investigated for application to rockets and spacecraft. Experimental data were obtained on the thermal conductivities of various materials in the cryogenic temperature range, as well as on the structural integrity and ablation characteristics of these materials at high temperatures occasioned by aerodynamic heating during atmospheric escape. Of the materials tested, commercial corkboard has the best over-all properties for the specific requirements imposed during atmospheric flight of a high-acceleration rocket vehicle.

From authors' summary

1683. White, J. A., A study of the guidance of a space vehicle returning to a braking ellipse about the earth, NASA TN D-191, 65 pp., Jan. 1960.

By assuming random errors in measurements of velocity, flight-path angle, thrust impulse, author analyzed guidance of space vehicle grazing earth atmosphere with specified perigee altitude. First-pass altitude was lowest allowable limited by heating, deceleration and accuracy of perigee altitude control. Trajectory required considerable accuracy. Errors in velocity and flight-path angle were viewed with limitations of instrumentation. Extreme accuracy requirements invited periodic thrust impulses. Author's study deals with portion of trajectory of vehicle from about 100,000 miles from earth's center to first-pass perigee altitude of 250,000 ft ± 25,000 ft. In all cases approaches followed elliptical path of eccentricity of almost one and at respective elliptic velocity. Assumptions: Earth-spherical; motion only in orbit's plane for non-rotating earth; gravitational fields of all other bodies negligible. Corrections were applied at intervals along path (Monte Carlo technique). Author used three methods of scheduling corrective thrust: (1) Certain decrease of radial distance from earth's center; (2) constant time intervals; (3) certain decrease of angle between perigee and vehicle position vector (30° and 45°).

Numerous plates of probability curves represent results. Each set is based on 1,100 runs; under certain conditions corrective values are read directly from graphs. Author provides discussion on effects of various factors. The three methods are compared, mostly with reference to perigee altitude. Among observations: Error in flight-path angle is predominant; final correction should be close to perigee; semimajor axis variation of initial trajectory (from 100,000 to 200,000 miles) had negligible effect; angular method provided excellent control.

V. A. Valey, USA

1684. Fisher, L. J., Jr., Landing energy dissipation for manned reentry vehicles, NASA TN D-453, 16 pp., Sept. 1960.

Analytical and experimental investigations have been made to determine the landing-energy-dissipation characteristics for several types of landing gear for manned reentry vehicles. The landing vehicles are considered in two categories: those having essentially vertical-descent paths, the parachute-supported vehicles, and those having essentially horizontal paths, the lifting vehicles. The energy-dissipation devices discussed are crushable materials such as foamed plastics and honeycomb for internal application in couch-support systems, yielding metal elements as part of the structure of capsules or as alternates for oleos in landing-gear struts, inflatable bags, braking rockets, and shaped surfaces for water impact.

It appears feasible to readily evaluate landing-gear systems for internal or external application in hard-surface or water landings by using computational procedures and free-body landing techniques with dynamic models. The systems investigated have shown very interesting energy-dissipation characteristics over a considerable range of landing parameters. Acceptable gear can be developed along lines similar to those presented if stroke requirements and human-tolerance limits are considered.

From author's summary

1685. Stillwell, W. H., and Larson, T. J., Measurement of the maximum speed attained by the X-15 airplane powered with interim rocket engines, NASA TN D-615, 14 pp., Sept. 1960.

1686. Stillwell, W. H., and Larson, T. J., Measurement of the maximum altitude attained by the X-15 airplane powered with interim rocket engines, NASA TN D-623, 12 pp., Oct. 1960.

1687. O'Keefe, J. A., The origin of tektites, NASA TN D-490, 26 pp., Nov. 1960.

Tektites are probably extraterrestrial, rather than the result of heating some terrestrial materials, because they are a chemically homogeneous group with definite peculiarities (high silica, excess of alkaline earths over alkalis, excess of potash over soda, absence of water), and because some of them (the australites) appear to have undergone ablation in flight through the atmosphere.

Since comparatively slow heating is required to explain the liquefaction of the tektite material, it is suggested that the tektites arrived along orbits which were nearly parallel to the surface of the earth, and which resulted from the decay of the orbit of a natural satellite. The great meteor procession of February 9, 1913, is an example of such an object. Comparison with the reentry phenomena of the artificial satellite 1957 Beta suggests that the 1913 shower consisted of a single large stone weighing about 400 kilograms, and a few dozen smaller bodies weighing about 40 grams each, formed by ablation from the larger body. It is shown that under the observed conditions considerable liquid flow would be expected in the stone, which would be heated to about 2100°K.

Objects falling from such a shower near the perigee point of the orbit would have a considerable distribution along the orbit as a result of slight variations in height or drag coefficient. The distribution in longitude would be made wider by the turning of the earth under the orbit during the time of fall.

The ultimate source of the body which produces a tektite shower is probably the moon, which appears, by virtue of its polarization and the phase distribution of the returned light, to contain high-silica materials.

It is suggested that the Igast object alleged to have fallen in 1855 is in fact genuine and represents an unmelted portion of the lunar crust.

From author's summary

1688. Sitte, K., Cosmic radiation problems in astronautics, *Bull. Res. Council Israel* **8** C, 1, 7-16, Feb. 1960.

Ballistics, Explosions

(See Revs. 1382, 1474, 1518, 1525, 1627, 1675)

Acoustics

(See also Revs. 1262, 1436, 1524)

1689. Rae, W. J., Potential flows in the approximation of viscous acoustics, AFOSR TN 60-409 (Cornell Univ., Grad. School Aero. Engng.), 107 pp., June 1960.

This paper investigates the effect of the small finite viscosity on the wave pattern and the loads of a body with and without lift. It demonstrates the dispersion of the conventional hyperbolic wave pattern and the rate of viscous decay of disturbances.

L. Trilling, USA

1690. Zhumartbaev, M. T., Absorption of sound and the width of shock waves in relativistic hydrodynamics, *Soviet Phys.-JETP* **10,** 4, 711-713, Apr. 1960. (Translation of *Zh. Eksp. Teor. Fiz., USSR* **37,** 1000-1004, Oct. 1959 by Amer. Inst. Phys., Inc., New York, N. Y.)

The intent of the author is to derive, in relativistic hydrodynamics, an absorption coefficient for sound wherein the coefficient is to be dependent upon viscosity and the nonrelativistic heat-conduction coefficient. The derivation is a straightforward exercise based upon a trio of Russian references. The result, however, is marred by misprints.

Thickness of shock waves is discussed in the usual manner with obvious relativistic modifications.

W. C. Orthwein, USA

1691. Morch, K. A., Measurement of total acoustic power of sources of sound in a reverberation chamber, *Acta Polytech. Scandinavica* PH 8, 25 pp., 1960.

Report summarizes the theory of the reverberant sound field, and an account of the reverberation chamber investigations at the laboratory is given. The reverberation chamber is to be used for measurement of the total acoustic power of sources of sound. Investigations have been carried out at frequencies between 2 and 15 kc/s.

From author's summary

1692. Wiener, F. M., Experimental study of the airborne noise generated by passenger automobile tires, *Noise Control* **6,** 4, 13-16, July/Aug. 1960.

1693. Kodaras, M. J., Reverberation times of typical elementary school classrooms, *Noise Control* **6,** 4, 17-19, July/Aug. 1960.

1694. Mahaffey, P. T., and Smith, K. W., Method for predicting environmental vibration levels in jet powered vehicles, *Noise Control* **6,** 4, 20-26, July/Aug. 1960.

Micromeritics

(See also Revs. 1422, 1500, 1501, 1513, 1514, 1624)

1695. Goniev, G. A., Questions of the dynamics of a granular medium (in Russian), Nauchn. Soobshch. Tsentr. Nauk-i. In-ta Stroit. Konstruktsii Akad. Str-va i Arkhitekt. SSSR no. 2, 1958, 122 pp. + illus.; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5605.

This is an attempt to evolve a theory for the plane motion of a granular medium. The system of equations for the motion, used by the author, consists of (1) two equations for the dynamic equilibrium in Euler variables for two coordinate axes, (2) conditions for Coulomb's boundary equilibrium, (3) conditions of incompressibility, and (4) conditions of convergence of the direction of the maximum velocities of shear deformation with the direction of the lines of slip. It should be noted at this point that conditions (3) and (4) contradict the presentation of the plastic potential as an entity. Equations are obtained representing four families of characteristics for this system of equations. A variant is given for the construction of an approximate theory for the granular medium, based on the modification of condition (4) on the basis of macrostructural concepts regarding the motion of the grains of the granular medium (in this case condition (4) appears to be an algebraical equation linking the components of the stress tensor with the values of the projection of the velocity vector onto the coordinate axis). A generalization is proposed for the equations to cover the case of a compressible medium, when the density of the medium at each point represents the function of the stressed state at that point. A series of quasidynamic problems is investigated (without taking into account the inertia forces) as possible examples of application of the theory: one-sided extrusion of the soil under a load, pressure of the soil on a supporting wall, steady motion of an incompressible granular medium in a vertical capacity (volumetric capacity?) and the dynamic problem on the irregular motion of a compressible granular medium in a vertical (volumetric?) capacity.

G. S. Shapito

Courtesy Referativnyi Zhurnal, USSR

1696. Avershin, I. N., Questions arising from separation processes when using mechanical classifiers (in Russian), *Obogashchenie Rud* no. 2, 26–31, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5292.

The classification of fine grains of sand is investigated as the deposition of semi-dispersal suspensions in a turbulent horizontal flow. It is also assumed that the velocity of the constricted descent ν depends on the diameter of the particles x for grains of identical density, according to the formula $V = K x^n$ where the coefficient K is determined by the specific gravity of the solid substance, the density and viscosity of the medium and other factors; n is dependent on the nature of the motion of the grains in the medium. In this case the separation into the fine class of particle size in units of one will comprise

$$\varepsilon_{-x+m} = \gamma \left(1 - \frac{k x_1^n}{(n+k) x_m^n}\right)$$

where γ – is the outlet to the classifier in units of one. The experimental verification of the equation showed its good convergence with the experiment.

U. Ts. Andres

Courtesy Referativnyi Zhurnal, USSR

1697. Manzhula, A. P., and Kalinin, I. A., The deposition of suspended particles from a turbulent flow (in Russian), *Inzh.-Fiz. Zh.* 1, 23–29, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5290.

The solution is examined of the equation for the convective diffusion (with constant coefficients) for a semi-space $z \geqslant 0$, when there is a certain point $z = b$ producing a momentary source of diffusional substance (with an assigned hydraulic coarseness) and in conditions of absorption of the substance on the boundary $z = 0$. The chief interest of the solution lies in the determination of the density of distribution of the absorbed substance on the boundary $z = 0$ which is found in the formula

$$C_1(x, y, \tau) = \int_0^\tau k_z \frac{\partial C}{\partial z}\bigg|_{z=0} dt$$

where C is the volumetric concentration of the substance with $z > 0$, determinable by means of the solution for the equation of diffusion. The unwieldy integral entering the expression for C_1 is analyzed and solved approximately. In particular, if the hydraulic coarseness of the particles being diffused, w, is small by comparison with the velocity of the flow, u, then at large distances, x, from the source downstream there will be an asymptotic presentation of the matter

$$C_1(x, y, \tau) \approx \frac{Qbu}{x^2} \varphi(w, b, \tau)$$

where Q is the intensity of the source.

A. S. Monin

Courtesy Referativnyi Zhurnal, USSR

1698. Bodryashkin, Ya. V., Hydraulic parameters at the beginning of the movement of sand deposits (in Russian), *Meteorol. i Gidrologiya* no. 10, 30–33, 1957; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5293.

The problem is examined regarding the determination of the critical values for the force of inducing motion of a flow S_0 at the beginning of the shifting of silt. Results of experiments are given which were carried out in a rectangular tank ($0.14 \times 0.15 \times 5$ m) with a variable slope, and in curvilinear and round tanks of the same section, and an analysis of the tests is furnished. Notwithstanding the small dimensions of the tanks, the accuracy of the experiments may be looked upon as satisfactory. An empirical formula is proposed for the force of inducing the flow.

I. I. Levi

Courtesy Referativnyi Zhurnal, USSR

1699. Donat, E. V., and D'yakov, V. V., Determination of the magnitude of the removal of pulverized material off the walls by a flow of air (in Russian), Industrial ventilation no. 6, Sverdlovsk, Metallurgizdat, 1957, 91–99; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5297.

This is a study of the removal of quartz dust from the lower horizontal wall of a tube when considering the capacity for dispersion of the dust and the velocity of the air flow. Special apparatus was used for the tests. This consisted of a conical tube of rectangular section (a confusor) 4.5 m long with an aperture angle of 4°, through which air was sucked. The value for the critical velocity, at which the removal of the dust commences, increases with diminution in size of the dust particles (fine dust possesses the quality to a large degree of nodulizing and clinging to the wall's surface). The magnitude of the removal of dust x in mg/m³ sec is linked with the velocity of the air flow ν by means of the following empirical equation

$$x = a(\nu - \nu_k)^n,$$

where ν_k is the critical velocity of the flow of air, a, n are constants, depending on the dispersion capacity of the dust.

L. A. Klyachko

Courtesy Referativnyi Zhurnal, USSR

1700. Mirzoeva, L. M., The experimental determination of the magnitude of sliding in a two-phase medium (gas-solid substance) during its motion in a vertical tube (in Russian), *Izv. Akad. Nauk AzerbSSR, Ser. Fiz.-Tekhn. i Khim. Nauk* no. 4, 87–92, 1958; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5303.

Results are given of the theoretical and experimental investigations undertaken to determine the linkage between the magnitudes of the coefficient for slip σ (the relation of the relative velocity of the motion of the solid particles U to the absolute velocity of the motion of gas in the vertical portion of tube C') and the actual (not the calculated) weight concentration of particles K' in the flow, expressing the relations of phases of a two-phase system (gas-solid substance) which is being carried along a vertical tube from the bottom upwards. The experiments were made with an alumosilicate powder, whose particles d had a mean diameter of 0.19 mm, in the following ranges of parameter changes: $0 \leqslant k' \leqslant 30$; the mean velocity of gas (air) C_m from 5 to 11 m/sec, and temperatures for the mixture of air with solid particles at the inlet to the experimental tube of 150 to 350°. The results of the experimental investigations are put forward in the form of graphical relations, σ to K' (for different C_m), σ to R_m and U to K' (for the relations of the density of the gas γ_r k to the density of the particles' material γ_p, equal to 0.333×10^{-3}, 0.289×10^{-3}, and 0.245×10^{-3}). Analysis of the graphs yielded the following conclusions: (1) In relation to the conditions of motion (the temperature of the gas and the velocity of the two-phase system) the coefficient of slip σ varies from 0.38 to 0.70; (2) the relative velocity of motion of particles U depends only feebly on the concentration of K' and increases with the increase in temperature of the carrier medium (i.e. decrease in the relation γ_r/γ_p); the magnitude of the coefficient of slip also depends only slightly on the concentration and increases with the decrease of the mean velocity of the carrier phase. Explanations are furnished for some discrepancies in the relations obtained solely on the basis of data of the experiments and with the aid of the calculation correlationships brought in.

Abstractor's note. The remarks relative to the influence of the number R_m or σ made by the author are not borne out by the graphs produced.

G. E. Khudyakov

Courtesy Referativnyi Zhurnal, USSR

1701. Ditiakin, U. F., and Britneva, L. N., Generalization of measurement results of drop sizes of the liquids pulverized by centrifugal atomizer with the assistance of dimensionless criteria (in Russian), *Teploenergetika* no. 11, 33-36, Nov. 1959.

1702. Ozdas, M. N., Oil transfer capacity of rotating disks, *Bul. Istanbul Tekn. Univ.* **12,** 1-14, 1959.

1703. Moskalev, N. M., The transportation of sludges of heavy consistency (in Russian), *Byul. Nauchno-Tekhn. Inform. po Gidrotekhn. i Melior.* no. 4, 27-29, 1957; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5140.

It is good economic practice to transport sludge with its natural content of water (or with the addition of a small quantity of water) by making use of its plastic-viscous properties. This reduces by 1.5-2 times the specific expenditure of energy required for its transport. The elastic properties of the sludge are dependent on the elasticity of the diffusion shells and the elasticity of the porous-cellular structure. While flowing, the structure of the sludge gradually breaks down and its elasticity is reduced. The elastic properties of a sludge with broken-down structures are reestablished in the course of time. Existing formulas for the determination of losses are not suitable for the case of sludge motion. A formula is proposed which resulted from the conversion of the known Buckingham formula which agrees with the results of the experiment carried out

$$I = 16\,\tau_0/3\,\gamma_n D + \lambda v^2/2\,g\,D,$$

where I is the hydraulic deviation, λ is the resistance coefficient in d'Arcy's known formula, γ_n the volumetric weight of the sludge, D the diameter of the sludge pipe, v the mean velocity of the flow, τ_0 the limit of resistance to shear. The first term of the formula is dependent on the elasticity of the sludge; the second, on the losses due to friction. The advantages of the formula are centered on its universal character (because of its applicability to any regimes of viscous or plastic-viscous liquids). The laboratory-commercial investigations of the flow of sludge along pipes were carried out over a wide range of velocities in pipes of different diameters. The experiment showed that at Reynolds figures less than 3000-4000 the flow of sludge possesses a plastic-viscous character, but at high figures the flow is turbulent. Coefficient λ is chosen from one or the other formula in relation to the regime.

B. M. Levin

Courtesy Referativnyi Zhurnal, USSR

Porous Media

(See also Revs. 1189, 1481, 1592, 1620, 1629)

1704. Gheorghita, St. I., Analogy between two-dimensional hydrodynamic and underground fluid flow (in Roumanian), *Gaz. Mat. Fiz.* (*A*) **10,** 9, 513-519, Sept. 1958.

Paper uses the well-known analogy between plane flows through homogeneous porous media and the potential plane motion of incompressible fluids. Proceeding from the velocity potential of a system consisting of a positive source and a negative one situated in the interior of a circle, author solves the problem of the determination of the rate of flow of a well situated in the interior of a semicircular boundary. The diameter is admitted as being the supply line on which the potential is constant and the semicircle is assumed to be impermeable.

It should be mentioned that there exist more general solutions for similar problems [see for instance; A. Hompeurt, "Eléments de mécanique des fluides dans les milieux poreux," Publications Techn., Paris, 1957, p. 110].

T. D. Oroveanu, Roumania

1705. Kirkham, D., Exact theory of flow into a partially penetrating well, *J. Geophys. Res.* **64,** 9, 1317-1327, Sept. 1959.

In this paper an exact theory is presented for the flow into a well that partially penetrates an artesian confined aquifer of uniform thickness. Author satisfies the Laplace equation and the boundary conditions by means of a series of Bessel functions.

Some of the coefficients of such functions are expressed as coordinates of an appropriate Fourier function. The other coefficients are determined through the solution of a system of algebraic linear equations, which author sets up by means of the boundary conditions. The number of these equations increases considerably with the ratio d/a of the depth d of the aquifer to the radius a of the well; this happens because the number of terms of the series, required to reach a good approximation, increases also with d/a. As soon as $d/a > 5\pi/4$ the numerical solution of the system requires much labor and becomes possible only with the help of a computer having a capacity of memory larger than that of the IBM 650 used by author. For $d/a \leq 5\pi/4$ the computations do not present particular difficulty, even when the ratio $b/a > 10$ (b distance between the external surface of the aquifer and the axis of the well); this happens thanks to a skillful simplification of the algebraic expressions.

Author works out numerical examples for values of $d/a = 5\pi/4$; $b/a = 10$ and for values of the ratio c/d (depth of penetration c to depth d of aquifer) 0; 1/5; 3/5; 1. Reviewer thinks that the study has a remarkable value from the mathematical point of view. However, a comparison of the results obtained by author with those deduced by current approximate procedures would have been helpful.

A. Russo Spena, Italy

1706. Christianovitch, S. A., Zheltov, Y. P., Barenblatt, G. I., and Maximovich, G. K., Theoretical principles of hydraulic fracturing of oil strata, Fifth World Petroleum Congress, 1959, Section II, Paper 23.

The origin and development of fissures during the process of hydraulic fracturing of a stratum are considered on the basis of certain physical hypotheses and of methods inherent to the theory of elasticity. Particular attention is paid to the case of the formation of horizontal and vertical fissures under conditions of constant rock pressure in the vicinity of the bore hole, and to the case of the formation of a horizontal fissure while said pressure is decreased because of the extrusion of plastic overlying layers into the bore hole.

Essentially three equations are given. The first one,

$$\frac{p-q}{\Delta P_c}\left(\frac{p-q}{E}\right)^3 = 0.0463\frac{(1+\nu^2)^2}{E}\frac{Q\mu}{Qt+\tau}$$

for a horizontal fissure, for example, enables one to calculate the ΔP_c, the pressure of the liquid at the base of the well minus the pressure of the liquid in the stratum. This equation involves the quantity q, the rock (or over-burden) pressure; the quantity E, Young's modulus of the rock; the quantity p, the well pressure; the viscosity of the fluid, μ; the injection rate, Q; the time of injection, t; and the volume of the breakdown fluid, τ. A similar equation is given for a vertical fracture.

A second equation,

$$V = 5(1-\nu^2)\frac{p-q}{E}R^3$$

enables one to calculate the radius of the fracture, R, knowing V, the volume of fluid injected into the fracture; and ν Poisson's ratio. No penetration of the liquid into the rock is assumed. This factor is, of course, of major importance as has been exampled by work which is used in the design of American fracturing treatments, see "Drilling and Production Practice," API 1957, Howard and Fast.

A third equation enables one to calculate the maximum width w_0 of the fracture. This equation is as follows:

$$w_0 = 0.513\frac{V}{R^2}$$

The Soviets have apparently employed hydraulic fracturing for water injection wells. They employ cellulose and paper industry waste as a fluid loss additive to reduce the penetration of the matrix by the fracturing fluid. E. W. Hough, USA

1707. Bourchardeau, A., and Rodier, J., A new method of determining the infiltration capacity of impermeable land (in French), *Houille Blanche* **15,** A, 521–527, July/Aug. 1960.

A difficulty encountered when preparing a flood hydrograph for a particular storm is that of estimating the infiltration capacity. A great number of factors are involved when determining the infiltration capacity of permeable land. An experimental determination of the amount of rain after which runoff begins makes it possible to reduce the problem to one of estimating the infiltration capacity of a heterogeneous surface, and this is much easier.

The Porcher method provides a solid basis from which to extrapolate infiltration capacities for heavy precipitation. This makes the forecasting of exceptional floods a much less random procedure. The method, which is applicable to land of medium and very great permeability, should be of much use where there is much vegetational cover or little runoff, conditions which frequently occur in temperate countries. From authors' summary

1708. Baryshev, V. M., Ibragimov, E. I., Malyshev, V. T., and Nordshtein, S. E., Alternate pumping into a stratum of sea water and alkaline water (in Russian), *Azerb. Neft. Kh-vo* no. 1, 12–15, 1954; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5332.

A mixture of equal volumes of sea and alkaline water possesses the same petroleum washing-out capacity when flooding strata as the more difficultly available alkaline water. The proposal is made to carry out the mixing of the waters in the stratum by means of alternately pumping in sea and alkaline water. For the purpose of study of the process of displacement of one type of water by the other experiments were made using an artificially compacted quartz sand bed to serve as the stratum. It was established, as the result of these experiments, that when the strata are being subjected to high pressures and temperatures there is no reduction in the permeability because no carbonates are precipitated; the velocity of filtration when sea water displaces the alkaline water is greater than in the reverse case; when one type of water displaces the other a mixing zone is created which extends with increase in the velocity of displacement and so forth.

A. M. Pirverdyan
Courtesy Referativnyi Zhurnal, USSR

1709. Shishchenko, R. I., and Avanesova, A. M., The influence of pressure and falls of pressure on the water yield from clay suspensions (in Russian), *Azerb. Neft. Kh-vo* no. 2, 14–17, 1955; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5333.

An investigation was carried out on the yield of water from clay suspensions, which had been treated with a waste alcoholic wash and alkali carbonate reagent, with pressure falls of up to 500 atm and also with changes in the pressure of up to 400 atm and a constant drop of pressure of 50 atm. The object of the investigation was to verify and add to the precision of the critical pressures established by Baranov [V. S. Baranov, "Water yield from clay suspensions, applicable when boring wells," *Grozn. Obl. Izd-vo,* 1947] for the water yield from clay suspensions treated as stated above. For solutions treated in this way a considerable drop in water yield with increase in the fall of pressure is confirmed. For solutions of the same nature a reduction of filtration capacity with increase of the pressure is established.

L. V. Lyutin
Courtesy Referativnyi Zhurnal, USSR

1710. Shekhtman, Yu. M., and Kuranov, I. F., The silting up of the zone adjacent to the face of feed wells (in Russian), *Azerb. Neft. Kh-vo* no. 5, 8–10, 1954; *Ref. Zh. Mekh.* no. 5, 1959, Rev. 5330.

A method is proposed for the calculation of complete wells which are being fed with a mixture of liquids of low concentration with suspended solid particles in them. Assumptions proposed are that the wells are spread over a horizontal stratum of constant thickness and that the influence of infiltration from neighboring wells can be disregarded. Besides, the authors make the further assumption that the discharge into the feed well does not alter with the passage of time. The equation for the silting-up process, based on a series of experimental data obtained in single-dimensional filtration of suspensions of low concentration and constant discharge, is taken to have the form of

$$\frac{\partial\zeta}{\partial t} = A\delta\frac{r}{r_c}(1-\zeta)$$

where ζ is the extent to which the pores have been blocked by deposited particles, δ is the suspension's concentration, r is the distance from the well's axis to any point in the stratum, r_c is the radius of the well, A is the experimental parameter. Approximate formulas for $\zeta(r, t)$ and $p(r, t)$—the pressure in the stratum—are obtained. An example of the calculation is given, but not of the comparison with the experimental data. The practical value of the conditions for the constancy of discharge in petroleum problems is somewhat doubtful. A. N. Patrashev

Courtesy Referativnyi Zhurnal, USSR

1711. Bozoky-Szeszich, K., Hydraulic aspects of furrow irrigation (in Hungarian), *Hidrológiai Közlöny* **40**, 1, 16–27, Feb. 1960.

1712. Torek, L., Methods for investigating non-steady percolation phenomena in an analogous model (in Hungarian), *Hidrológiai Közlöny* **40**, 1, 28–32, Feb. 1960.

1713. Ollos, G., The effect of inhomogeneous soils on well discharge (in Hungarian), *Hidrológiai Közlöny* **40**, 1, 33–47, Feb. 1960.

Geophysics, Hydrology, Oceanography, Meteorology

(See Revs. 1190, 1433, 1515, 1533, 1535)

Naval Architecture and Marine Engineering

(See also Revs. 1411, 1663)

1714. Rosingh, W. H. C. E., and Berghuis, J., Mathematical shipform, *Inter. Shipbldg. Prog.* **6**, 53, 37–42, Jan. 1959.

Authors present an approximate analytic method which permits computation of ship lines as well as displacement and hydrostatic curves. The method has been programmed for use on the electronic computer Gamma E.T. As an illustrative example, the method was applied to the determination of ship lines for the passenger liner "Nieuw Amsterdam."

Careless translation detracts somewhat from the clarity of the paper. B. V. Nakonechny, USA

1715. Kaplan, P., and Hu, P. N., Three-dimensional stripwise damping coefficients for heave and pitch of a submerged slender spheroid, *J. Ship Res.* **4**, 1, 1–7, June 1960.

Zero forward speed is assumed, viscosity is neglected, and the damping is assumed to be due to waves generated by the oscillating body. The slender spheroid is represented by a distribution of vertical dipoles along the axis, of local strength proportional to the local cross section and the local vertical velocity. The total energy radiated by this distribution can be calculated from the asymptotic value of the velocity potential at a great distance. The authors consider whether it would be useful to interpret this total energy in terms of a strip theory, and show that consistent results are obtained with suitable definitions of local energy radiation for heave and pitch which, however, depend on the distribution of area along the whole length, not only on the local area. The authors next consider the expressions for force and moment in terms of an integral over the dipole strength, in the useful form due to W. Cummins [*J. Ship Res.* **1**, 7–18, 1957; AMR **10**(1957), Rev. 4063] and ask whether it would be useful to interpret the integrands as local elementary forces and moments which, when multiplied by the appropriate velocity components, would give the energy carried away by waves. This requires a more detailed consideration of the image potential and leads to expressions for elementary forces and moments very similar to those given by the first method. These are evaluated for the slender spheroid, and it is concluded that important three-dimensional effects are concealed by the simple two-dimensional strip theory.

F. Ursell, England

1716. Gerritsma, J., Ship motions in longitudinal waves, *Inter. Shipbldg. Prog.* **7**, 66, 49–71, Feb. 1960.

Frequency response functions of ship models in longitudinal waves are determined by three methods and results are compared. The first method consists of measuring heave and pitch in regular waves. The second method requires the experimental determination of the coefficients of coupled second-order differential equations describing the motion. The third method analyzes test results obtained in irregular waves. The models used were members of the series sixty family. The behavior of the 0.60, 0.70 and 0.80 block coefficient models are discussed in detail along with the limitations of the linearized theory, with the effect of coupling, etc. Agreement (except in special cases) is good among the results found by the three methods.

Paper is a valuable contribution.

V. G. Szebehely, USA

1717. Kaplan, P., Breslin, J. P., and Jacobs, Winnifred R., Evaluation of the theory for the flow pattern of a hydrofoil of finite span, *J. Ship Res.* **3**, 4, 13–29, Mar. 1960.

Paper is a review of available theoretical results for two- and three-dimensional hydrofoils based upon potential theory. Expressions derived for the surface-wave elevation and downwash are compared with limited experimental values for a tandem hydrofoil system of aspect ratio 20. The three-dimensional effects are shown to be rather great but represented fairly well by the recommended theoretical formulas. S.-F. Shen, USA

1718. Moran, J. P., An equivalence principle for water-exit and -entry problems, *J. Aerospace Sci.* **27**, 11, 876–877 (Readers' Forum), Nov. 1960.

Friction, Lubrication and Wear

(See also Revs. 1373, 1417)

1719. Ganesha Rao, H. M., A study of journal bearing, Part 2, *J. Instn. Engrs., India* **40**, 7 (Part 2, Mechanical, Electrical & General Engineering), 253–257, Mar. 1960.

The paper makes use of various fundamental relations, given in the literature, which make possible the inclusion of the effects of viscosity dependence on temperature and pressure on the film temperature. The journal bearing is considered and the condition of no end leakage is assumed. Heat loss through conduction is not included and the relations thereby represent an upper bound to temperature rise. It would be interesting to know the degree to which the results would be borne out by experiment.

F. R. Archibald, USA

1720. Gross, W. A., Steady performance characteristics of gas-lubricated journal bearings with slenderness ratio $L/D = \pi$, *J. Aerospace Sci.* **27**, 11, 869–870 (Readers' Forum), Nov. 1960.

1721. Nica, A., Considerations on the kinematics of roller bearings in case of hydrodynamic lubrication (in English), *Rev. Mech. Appl.* **4**, 3, 463–468, 1959.

Paper discusses friction between rolling bodies and raceways of roller and needle bearings under a constant radial load. Equations for friction moment derived in previous papers, unavailable to this writer, are used. Solutions to these equations are obtained in this paper. W. J. Anderson, USA

1722. Avient, B. W. E., Goddard, J., and Wilman, H., An experimental study of friction and wear during abrasion of metals, *Proc. Roy. Soc. Lond. (A)* **258**, 1293, 159–180, Oct. 1960.

1723. Sliney, H. E., Lubricating properties of some bonded fluoride and oxide coatings for temperature to 1500° F, NASA TN D-478, 26 pp., Oct. 1960.

The lubricating properties of some experimental ceramic coatings, diffusion-bonded fluoride coatings, and ceramic-bonded fluoride coatings were determined. The experiments were conducted in an air atmosphere at a sliding velocity of 430 feet per minute and at temperatures from 75° to 1500° F.

Several ceramic coatings provided substantial reductions in friction coefficient and rider wear (compared with the unlubricated metals). For example, a cobaltous oxide (CoO) base coating gave friction coefficients of 0.24 to 0.36 within the temperature range of 75° to 1400° F; serious galling and welding of the metal surfaces were prevented. The friction coefficients were higher than the arbitrary maximum (0.2) usually considered for effective boundary lubrication. However, when a moderately high friction coefficient can be tolerated, this type of coating may be a useful antiwear composition. Diffusion-bonded calcium fluoride (CaF_2) on Haynes Stellite 21 and on Inconel X gave friction coefficients of 0.1 to 0.2 at 1500° F. Endurance life was dependent on the thermal history of the coating; life improved with increased exposure time at elevated temperatures prior to running. Promising results were obtained with ceramic-bonded CaF_2 on Inconel X. Effective lubrication and good adherence were obtained with a 3 to 1 ratio of CaF_2 to ceramic. A very thin sintered and burnished film of CaF_2 applied to the surface of this coating further improved lubrication, particularly above 1350° F. The friction coefficient was 0.2 at 500° F and decreased with increasing temperature to 0.06 at 1500° F. It was 0.25 at 75° F and 0.22 at 250° F.

A survey of the thermochemical properties of the halides was of considerable aid in selecting metal halides that might be chemically stable in air at 1500° F. Good correlation was obtained between predictions based on thermodynamics and the results of experimental thermal stability studies.

From author's summary

1724. Buckley, D. H., and Johnson, R. L., Halogen-containing gases as lubricants for crystallized-glass-ceramic-metal combinations at temperatures to 1500° F, NASA TN D-295, 19 pp., Oct. 1960.

Pyroceram 9608 (a crystallized glass ceramic) has been considered for use in high-temperature bearing and seal applications. One of the problems encountered with Pyroceram is the lack of availability of lubricants for the temperature range in which this material becomes practical. Experiments were conducted with Pyroceram sliding on various nickel- and cobalt-base alloys using reactive halogen-containing gases as lubricants. Friction and wear data were obtained as a function of sliding velocity and temperature.

Studies were made with a hemispherical rider (3/16-in. rad, Pyroceram 9608) sliding in a circumferential path on the flat surface of a rotating disk (2½-in. diam, nickel- or cobalt-base alloys). The specimens were run in an atmosphere of the various gases with a load of 1200 grams, a sliding velocity of 3200 feet per minute, and temperatures from 75° to 1500° F.

The gas CF_3Br—CF_2Br was found to be an effective lubricant for Pyroceram 9608 sliding on Hastelloy R-235 and Inconel X up to 1400° F. The gas CF_2Cl—CF_2Cl provided effective lubrication for Pyroceram sliding on various cobalt-base alloys at 1000° F.

From authors' summary

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